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Pace of Tower Shadow Fluctuations in a Wind Farm

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***Abstract*— This work focuses on the overall effect of blade positions in a wind farm. Assuming that there are no synchronization forces in a wind farm, the distribution of blade position of the turbines of a wind farm is derived. The crossing of blades in front of its turbine tower is modeled as a Poisson Process. The variability of the wind farm power output due to tower shadow and the probability of extreme conditions (such as simultaneous tower shadow events at all turbines of a wind farm) are estimated in the time domain.**

***Index Terms*—wind power, flicker, power fluctuation**

I. INTRODUCTION

Fast fluctuations of power output can be divided into cyclic components (tower shadow, wind shear, modal vibrations, etc.), weather dynamics and events (connection or disconnection of the turbine, change in generator configuration, etc.).

The influence of blade position in a single turbine power output has been widely analyzed in the literature [1]. According to [2], very steady wind, very uniformly distributed and a weak electrical network is necessary for the synchronisation of blade position on the turbines of a farm. Experimental measures [3] showed that the synchronisation of blades is unusual.

If the turbines blades are not synchronized, the cyclic uncorrelated fluctuations due to rotor position have random phases. As wind characteristics are similar inside the farm, the magnitude of the cyclic components would be similar in all turbines. The phase difference among turbine blades i and j is $\varphi_{i,j}$ and it is uniformly distributed in $[-\pi, +\pi]$. Therefore, phase difference of harmonic k in turbines i and j is $k\ \varphi_{i,j}$.

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II. WIND FARM PERIODIC FLUCTUATIONS IN THE TIME DOMAIN

The power dips due to blade position will be considered as negative power pulses distributed uniformly in time $t \in [0, T]$, where T is the period of the pulses (the period in Fig. 1 is T = 1s). For a turbine of three blades, the period in seconds is $T = 20 / \Omega_{rotor}$, where Ω_{rotor} is the rotor in r.p.m. and the conversion factor is 60 s/min divided by 3 blades in the rotor, equal to 20 blades/r.p.m. The effective period of a wind farm – whose turbines can have different Ω_{rotor} – is the average of the periods of its turbines.

Since the occurrence of pulses is not correlated, usual techniques for the computation of the sum of identically distributed independent random variables are applicable. They involve iterative computation of convolutions or inverse Fourier transform of the characteristic function to the power N.

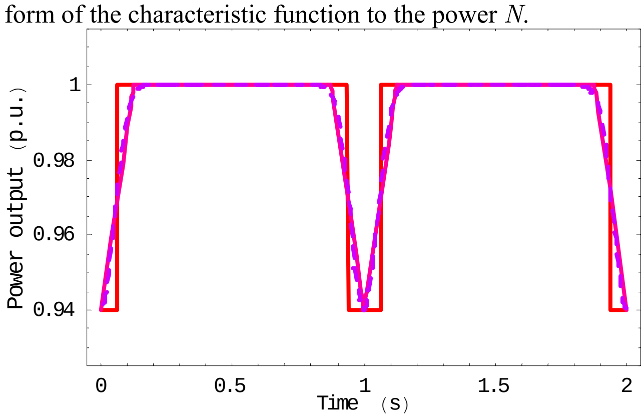

Fig. 1: Power output at a single turbine with blade rate 1 Hz, depth of tower shadow α = 0,06 p.u. and average power loss = $\alpha\, \tau / T$ = 0,0075 p.u. (the power dip shapes are rectangular, triangular and Gaussian).

Tower shadow of shapes rectangular, triangular, cosine and Gaussian have been compared to test the shape influence in overall behavior of the farm.

$$\mathrm{P_{turbine}(t)} = \mathrm{P_0} - \mathrm{f_{pulse}(t)}; \qquad (1)$$

$$\mathrm{f_{pulse\Delta}(t)} = \begin{cases} \alpha\left(1-\dfrac{|\mathrm{t}-\mu|}{\tau}\right) & |\mathrm{t}-\mu| < \tau \\ 0 & |\mathrm{t}-\mu| \geq \tau \end{cases} \quad (\tau < T/2) \qquad (3)$$

$$\mathrm{f_{pulse\Box}(t)} = \begin{cases} \alpha & |\mathrm{t}-\mu| < \tau/2 \\ 0 & |\mathrm{t}-\mu| \geq \tau/2 \end{cases} \quad (\tau < T) \qquad (4)$$

$$\mathrm{f_{gaussian\ pulse}(t)} = \alpha\, e^{-\left(\sqrt{\pi}(t-\mu)/\tau\right)^2} \quad (\sigma = \tau/\sqrt{2\pi}) \qquad (2)$$

$$\mathrm{f_{cos\ pulse}(t)} = \begin{cases} \dfrac{\alpha}{2}\left[1+\cos\left(\dfrac{\pi(t-\mu)}{\tau}\right)\right] & |\mathrm{t}-\mu| < \tau \\ 0 & |\mathrm{t}-\mu| \geq \tau \end{cases} \qquad (5)$$

$$Max(\mathrm{f_{pulse}(t)}) = \alpha; \qquad (6)$$

$$\textit{Pulse Energy Loss} = \int_{\mu-T/2}^{\mu+T/2} \mathrm{f_{pulse}(t)}dt = \alpha\,\tau \qquad (7)$$

$$\textit{Average Power Loss} = \frac{\textit{Pulse Energy Loss}}{\textit{time between pulses}} = \alpha\,\frac{\tau}{T} \qquad (8)$$

where α is the depth of the pulse, τ is the characteristic width and μ is the time position of the pulse. For convenience, the origin of time will be chosen in the center of the pulse (μ=0). Tower shadow duration is from 1/8 (according to [1]) up to the tower shadow period and α depends greatly on inertia, type and control of the generator.

A. Experimental tower shadow at a $\mathrm{P_0}$ *=750 kW wind turbine (squirrel cage induction generator and stall regulation).*

The tower shadow shape in the turbines with squirrel cage induction generator and stall regulation has been measured by the authors at Valdecuadros wind farm [3]. Fig. 2 shows the typical behavior for wind speeds around 6,5 m/s. Tower shadow resembles a sinusoidal fluctuation of average amplitude modulation around $\mathrm{P_0}/15$ superposed to slower fluctuations due to wind evolution.

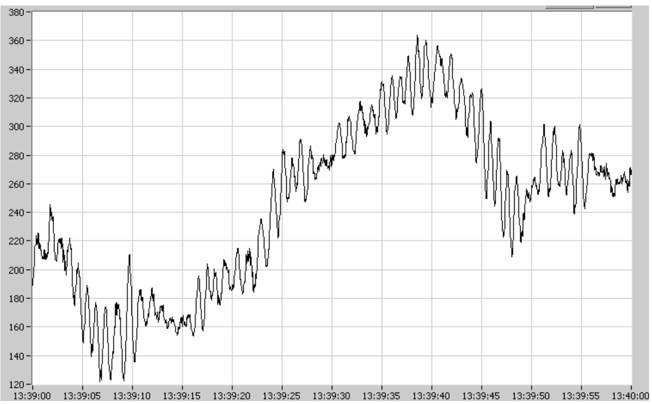

Fig. 2: Active power of a NTM 750 kW wind turbine for wind speeds around 6,5 m/s during one minute.

For simplicity, it can be characterized as a sinusoidal fluctuation at the blade frequency with random amplitude. However, the shape and the amplitude vary and they are quite random. To test this, the power spectrum density (PSD) of power during 5 minutes have been calculated [4], showing an exponential decay (a line in the log-log plot) for wind and power, superposed to a wide amount of spectral fluctuation around tower shadow frequency $1/T$, rotor frequency $1/(3T)$ and half blade frequency $2/(3T)$. The harmonics of tower shadow are very sharp but their power content is much lower than the fundamental component. Therefore, tower shadow harmonics can have structural concerns but their associated power is small.

B. Distribution of the fluctuation

The first step is the computation of the distribution of the pulse. If the pulse is symmetrical about its mass center, its PDF is the inverse function divided by half the pulse energy. Then, the CDF can be computed, taking into account the symmetry of the pulse and taking the positive branch (t >0) of the inverse function of $f_{pulse}(t)$.

$$\mathrm{CDF_{pulse}}(y) = \Pr[\mathrm{f_{pulse}}(t) < y] = \\ = \Pr[t > \mathrm{f_{pulse}}^{-1}(y) | 0 < t < \mathrm{T}/2] \qquad (9)$$

Assuming $0 < t < \mathrm{T}/2$, $0 \leq y \leq \alpha$ and μ=0 (symmetric pulse centered in time origin), the general formula is:

$$\mathrm{CDF_{pulse}}(y) = 1 - \frac{\mathrm{f^{-1}}_{\mathrm{pulse}|\mu=0}(y)}{T/2} \qquad (10)$$

From (10), the distributions of fluctuations of a single turbine due to triangular, rectangular and Gaussian pulses are:

$$\mathrm{CDF_{pulse\Delta}}(y) = 1 - \frac{\tau(1-y/\alpha)}{T/2} \\ \Rightarrow \mathrm{PDF_{pulse\Delta}}(y) = \frac{2\,\tau}{\alpha\,T} + \left(1 - \frac{2\,\tau}{\alpha\,T}\right)\delta(y) \qquad (11)$$

$$\mathrm{CDF_{pulse\Box}}(y) = \left(1-\frac{\tau}{T}\right)U(\mathrm{y}) + \frac{\tau}{T}U(\mathrm{y}-\alpha) = \begin{cases} 1 & y \geq \alpha \\ 1-\dfrac{\tau}{T} & 0 \leq y < \alpha \end{cases}$$

$$\Rightarrow \mathrm{PDF_{pulse\Box}}(y) = \left(1-\frac{\tau}{T}\right)\delta(y) + \frac{\tau}{T}\delta(y-\alpha) \qquad (12)$$

$$\mathrm{CDF}_{\underset{gaussian}{\mathrm{pulse}}}(y) = 1 - \frac{\tau}{T/2}\sqrt{\frac{Ln(\alpha/y)}{\pi}} \\ \Rightarrow \mathrm{PDF}_{\underset{gaussian}{\mathrm{pulse}}}(y) = \frac{\tau}{T\,y\sqrt{\pi Ln(\alpha/y)}} \qquad (13)$$

where U states for the Heaviside unit step function and δ for the delta of Dirac.

The characteristic function $\phi(w)$ is the complex conjugate of the continuous Fourier transform of $\mathrm{PDF_{pulse}}(y)$:

$$\phi_{turbine}{}^{*}(w) = \int_0^{\alpha} e^{-j\,w\,y}\,\mathrm{PDF_{pulse}}(y)\,dy \qquad (14)$$

Since the tangent of the pulse is horizontal at some points (at least at the maximums and minimums), $PDF_{\mathrm{pulse}}(y)$ contains essential singularities and its transform should be computed analytically.

The pulsating power of a wind farm with N turbines is the sum of individual power pulses, supposed identically distributed independent random events. The corresponding characteristic function is $\phi_{turbine}{}^{N}(w) = \phi_{wind\ farm}(w)$. Thus, the PDF of the wind farm deviation can be computed as inverse Fourier transform of the complex conjugate of $\phi_{wind\ farm}(w)$:

$$PDF_{\Delta P wind\ farm}(y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{j\,w\,y}[\phi_{turbine}{}^{*}(w)]^{N}\ dw \quad (15)$$

The characteristic function of a wind farm with rectangular power dips shown at Fig. 1 corresponds to a binomial distribution. This makes sense since the number of simultaneous pulses in a wind farm is the probability of the number of successes in a sequence of N independent "*pulse*" / "*no pulse*" experiments with success probability $p = \tau / T$ (the relative width of the pulse). Therefore, the instantaneous fluctuation in the wind farm follows a Binomial distribution:

$$PDF_{\Delta P wind\ farm\square}(y) = \binom{N}{y/\alpha}\left(\frac{\tau}{T}\right)^{y/\alpha}\left(1-\frac{\tau}{T}\right)^{N-y/\alpha} \quad (16)$$

$$CDF_{\Delta P wind\ farm\square}(y) = I_{1-\tau/T}(N - y/\alpha,\ y/\alpha + 1) \quad (17)$$

where $I_{1-\tau/T}$ is the regularized incomplete beta function.

The average tower shadow effect in a wind farm is $\langle y\rangle = \alpha\langle y/\alpha\rangle = \alpha N p$. Its standard deviation is:

$$\sigma_y = \alpha\ \sigma_{y/\alpha} = \alpha\sqrt{N\,p\,(1-p)} = \alpha\frac{\tau}{T}\sqrt{N}\sqrt{\frac{\tau}{T}-1} \quad (18)$$

which scales with $\sqrt{N}$ instead of being proportional to the number of turbines N.

Fig. 3 shows the CDF of the relative deviation of power output at a wind farm due to tower shadow effect. *Pulse sum (p.u.)* is $y/\alpha N$, the ratio of the experienced power deviation y relative to the maximum deviation, αN, which corresponds to all tower shadows happening simultaneously.

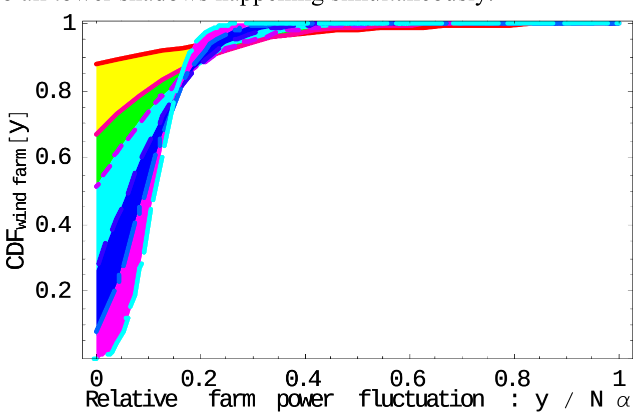

Fig. 3: CDF of pulses at a wind farms of 1, 3, 5, 10, 20 and 50 turbines (starting from upper part at zero fluctuation). The parameters of the pulses correspond to Fig. 1 and rectangular shape ($p = \tau/T = 0{,}125$). The discrete CDF (17) has been evaluated at midpoints to account that real pulses are continuous and derivable.

The probability of simultaneous tower shadows events at most turbines is very low, as can be seen in the figure, $\mathrm{CDF}_{\mathrm{pulse}}(3\langle y\rangle) \approx 1$. The probability of exceeding a certain amount of power dip is the complementary CDF:

$$\Pr[\Delta P_{\substack{wind\\farm}} > y] = 1\text{-}\mathrm{CDF}_{\Delta P_{\substack{wind\\farm}}}(y) = I_{\frac{\tau}{T}}\left(\frac{y}{\alpha}+1,\ N-\frac{y}{\alpha}\right) \quad (19)$$

The average power loss due to tower shadow is $\langle y\rangle$ (there are exactly N pulses in a period T).

$$\langle y\rangle = \frac{N}{T}\int_{-T/2}^{+T/2} \mathrm{f}_{\substack{\text{single}\\\text{pulse}}}(t)\ dt = \frac{N}{T}\alpha\,\tau \quad (20)$$

Since turbine rotor angles are independent random variables, the variance of wind farm power output due to turbine blade position, $\sigma_y{}^2$, can be computed as: (21)

$$\sigma_y{}^2 = N\left[\frac{1}{T}\int_{-T/2}^{+T/2} \mathrm{f}_{\substack{\text{single}\\\text{pulse}}}{}^2(t)\ dt - \left(\frac{1}{T}\int_{-T/2}^{+T/2} \mathrm{f}_{\substack{\text{single}\\\text{pulse}}}(t)\ dt\right)^2\right]$$

C. *Rate of tower shadow events*

The alignments of blades with their tower axis are [tower shadow] events whose time occurrence can be modeled as a stochastic process. The number of tower shadow events in the period T is the number of turbines in a wind farm, N.

The event rate or event *intensity* $\lambda(t)$ is the number of events per unit time and its average can be computed as $\lambda_0 = \langle\lambda(t)\rangle = N/T$ since in one period exactly one blade of all turbines of the wind farm passes in front of its tower (provided all turbines were spinning approximately at the same speed). The event rate λ_0 can be thought of as the probability that a blade alignment occurs in a specified interval.

1) *Prior probability distributions*

Since there is no explicit time origin and there are no appreciable synchronizing forces, the event can occur at any instant with the same likelihood and λ_0 is constant. This implies that the time between consecutive events (called interarrival times) are independent random variables. The only interarrival time distribution with constant hazard rate is the exponential. The waiting time Δt until the first occurrence is a continuous random variable with an exponential distribution (with parameter λ_0). This probability distribution may be deduced from the fact that

$$\Pr(\Delta t_{interarrivial} > t) = \Pr(N_{events} = 0\ in\ interval\ (0,t]) =$$
$$= e^{-\lambda_0 t} = 1 - CDF_{Exponential(\lambda_0)}(t) \quad (22)$$

For exponentially distributed events, the Poisson distribution is the probability distribution of the number of events that would occur within a preset time t.

$$\Pr(N_{events} = k\ in\ interval\ (0,t]) =$$
$$= PDF_{Poisson(\lambda_0 t)}(k) = \frac{e^{-\lambda_0 t}(\lambda_0 t)^k}{k!} \quad (23)$$

The Erlang distribution describes the waiting time until k tower shadow events have occurred when inter-event time is distributed exponentially. The probability density function of the Erlang distribution is (24)

$$\Pr(\Delta t_{waiting} \le t \mid N_{events} = k) = CDF_{Erlang(k,\lambda_0)}(t) = \frac{\gamma(k,\lambda_0 t)}{(k-1)!}$$

where $\gamma(\cdot)$ is the lower incomplete gamma function. The probability density is:

$$PDF_{Erlang(k,\lambda_0)}(t) = \frac{\lambda_0{}^k t^{k-1} e^{-\lambda_0 t}}{(k-1)!} \quad (25)$$

2) Including periodicity in probability distributions

During normal operation, turbine speed fluctuates slightly. Multi-megawatt turbines spin at Ω_{rotor} = 8~20 rpm, implying blade periods T = 1~2,5 s. During a short time interval, the turbine speed can be considered constant and the time interval between two consecutive tower shadow events of the same turbine is (approximately) T. If all turbines of a wind farm rotate at the same approximate speed Ω_{rotor} , each turbine must experience one and only one tower shadow event in the interval (0, T] with uniform probability.

The implications of periodicity can be included in the probability of the number of tower shadow events in a time interval Δt using Bayes' theorem, provided $0 \leq \Delta t < T$ and $0 \leq k \leq N$:

$$\Pr(N_{events} = k \ in \ (0,t] \mid N_{events} = N \ in \ (0,T]) = \quad (26)$$

$$= \frac{\Pr(N_{events}{=}k \ in \ (0,t])\cdot\Pr(N_{events}{=}N \ in \ (0,T] \mid N_{events}{=}k \ in \ (0,t])}{\Pr(N_{events} = N \ in \ (0,T])}$$

$$= \frac{PDF_{Poisson(\lambda_0\, t)}(k)\cdot PDF_{Poisson(\lambda_0\,(T-t))}(N-k)}{PDF_{Poisson(\lambda_0\, T)}(N)} =$$

$$= \binom{N}{k}\left(\frac{t}{T}\right)^{k}\left(1-\frac{t}{T}\right)^{N-k}$$

The PDF of the number of tower shadow events in an interval t is a binomial distribution of N trials, k events and event probability $p = t/T$. (This equation is equivalent to (16), where the pulse width τ has been replaced by the interval time t, and the depth ratio of the power dip at the farm y/α has been replaced by the number of pulses, k).

The probability density of the waiting time t_k until the k^{th} occurrence can be computed using Bayes' theorem, provided $0 < t < T$ and $0 < k < N$:

$$\mathrm{PDF}(\,waiting\ time\, t\ for\ k\ events \mid N_{events} = N\ in\ (0,T]) =$$

$$= \frac{PDF_{\substack{waiting\ time\\ for\ k\ events}}(t)\cdot PDF_{\substack{waiting\ time\\ for\ N-k\ events}}(T-t \mid N_{events}{=}N\ in\ (0,T])}{\mathrm{PDF}_{\substack{\text{waiting time}\\ \text{for N events}}}(T)}$$

$$= \frac{PDF_{Erlang(k,\lambda_0)}(t)\cdot PDF_{Erlang(N-k,\lambda_0)}(T-t)}{PDF_{Poisson(\lambda T)}(N)} =$$

$$= \frac{N-1}{T}\binom{N-2}{k-1}\left(\frac{t}{T}\right)^{k-1}\left(1-\frac{t}{T}\right)^{N-k-1} \quad (27)$$

The PDF of the waiting time t resembles the binomial distribution of N-2 trials, k-1 successes and success probability p = t/T, multiplied by a normalizing factor.

D. Modulation of the pulse density at the wind farm with randomly distributed pulses.

The interarrival time between pulses k and k+1 will be denoted $\Delta t(k) = \Delta t_{k,k+1}$. The interarrival times are not constant, but it has a mean value $\langle \Delta t(k)\rangle = \Delta t_0 = T/N$.

The expected number of tower shadow occurrences during the time unit is the inverse of the mean interarrival time, $\lambda_0 = 1/\Delta t_0 = N/T$ and λ_0 is also the average frequency of occurrence, measured in hertz, and the average blade rate of the wind farm. At an instant t between pulses k and k+1, the instantaneous frequency $\lambda(t)$ of tower shadow events at the wind farm output can be computed as $\lambda(t) = 1/\Delta t_{k,k+1}$. In Poisson process theory [5], the event rate $\lambda(t)$ is the parameter of the process, whereas the interarrival time $\Delta t(k)$ is an outcome of the process. When the number of wind turbines is big ($N > T/\tau$ or $\lambda_0\ \tau > 1$), the density of blade events $\lambda(t)$ is more significant than the interarrival time $\Delta t_{k,k+1}$.

At an instant between pulses k and k+1, the instantaneous angular frequency w_1 of tower shadow at wind farm output can be computed as $w_1(t) = 2\ \pi\ \lambda(t) = 2\ \pi\ /\ \Delta t_{k,k+1}$. The angular frequency will oscillate around its average value $\langle w_1(t)\rangle = 2\pi\ \lambda_0 = 2\pi N/T$.

When $N > T/\tau$ (or equivalently $\lambda_0\ \tau > 1$), the effects due to the sharp shape of the pulse diminish and the main contribution to wind farm fluctuations is due to the possible concentration of tower shadow events in a part of the period T.

In a real wind farm, the pulse rate λ is not constant in the period. Fundamental harmonic (m = 1) measures how much the pulses are concentrated in half period. The order 2 harmonic (m = 2) measures if the tower shadow events occurs preferably every T/4 seconds.

The event density at a given time t is $\lambda(t)$ (this will be used in the next subsection for computing the modulation of power output).

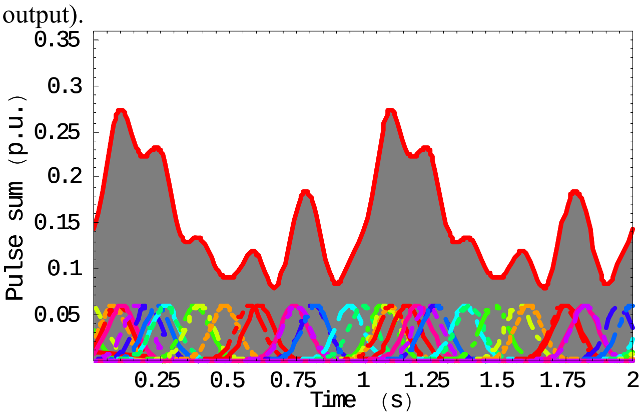

Fig. 4: Individual tower shadow pulses with Gaussian shape and random blade position corresponding to a wind farm with N = 20 turbines spinning at Ω = 20 rpm (1 blade/s).

1) Estimation of coefficients of Fourier series of random pulses

The rate of tower shadow pulses is the number of blades divided by the mean rotor speed, $\lambda = blades\,/\,\Omega_{rotor}$. The instantaneous power due to tower shadow and wind shear has period $T = 1/\lambda$. In general, the pulse is not centered at the time origin and the Fourier coefficients will be complex numbers except the DC component (term of zero order).

If turbine speeds are equal, power is cyclic with period T and there are N tower shadow events in each cycle. Therefore, power can be decomposed in its Fourier series of harmonics of fundamental angular frequency $w_1 = 2\pi/T$. As power is a stochastic magnitude, the coefficients of Fourier series are stochastic complex values (coefficients are not real since instantaneous power is not symmetric with respect to origin).

$$P_{farm}(t) = \frac{c_0}{2} + \sum_{m=1}^{\infty} \mathrm{Re}\left[\overrightarrow{c_m}\, e^{j\,m\,w_1 t}\right] \quad (28)$$

The distribution of the complex Fourier coefficients $\overrightarrow{c_m}$, $m \neq 0$, can be estimated taken into account that:

- Fourier transform is linear and, thus, the transform of wind farm output is the sum of the transform of the individual turbines:

$$\boldsymbol{\mathcal{F}}\left\{\sum_{i=1}^{N} \mathrm{f}_{\substack{\text{pulses}\\ \text{turbine } i}}(t-\mu_i)\right\} = \sum_{i=1}^{N} \boldsymbol{\mathcal{F}}\left\{\mathrm{f}_{\substack{\text{pulses}\\ \text{turbine } i}}(t-\mu_i)\right\}$$

- The Fourier coefficient for a single turbine whose tower shadow event coincides with time origin can be obtained from $F_{\text{single pulse}}(w)$, the continuous Fourier transform of a single pulse centered in time origin.

$$\overrightarrow{c_m}\Big|_{\substack{\text{single turbine}\\ \text{with pulses at}\\ t=0,T,2T,3T\ldots}} = 2\,\lambda\, F_{\substack{\text{single}\\ \text{pulse}}}\left(\frac{2\pi m}{T}\right) = \frac{2}{T} F_{\substack{\text{single}\\ \text{pulse}}}\left(\frac{2\pi m}{T}\right) \quad (29)$$

- The tower shadow can occur at turbine i at any time μ_i with equal probability. Hence μ_i is uniformly distributed in the period $[0, T]$.
- The circular time shift property of Fourier transform implies that $\boldsymbol{\mathcal{F}}\{\mathrm{f}_{\text{pulses}}(t-\mu_i)\} = e^{-jw\,\mu_i}\boldsymbol{\mathcal{F}}\{\mathrm{f}_{\text{pulses}}(t)\}$, where $\mathrm{f}_{\text{pulses}}(t)$ is the pulse train centered in the time origin, $\mathrm{f}_{\text{pulses}}(t-\mu_i)$ is the pulse train displaced μ_i time and w is the angular frequency.

$$\boldsymbol{\mathcal{F}}\left\{\mathrm{f}_{\substack{\text{pulses}\\ \text{turbine } i}}(t-\mu_i)\right\} = e^{-jw\,\mu_i}\boldsymbol{\mathcal{F}}\left\{\mathrm{f}_{\substack{\text{pulse train centered}\\ \text{in time origin}}}(t)\right\} \quad (30)$$

$$\Rightarrow \left|\boldsymbol{\mathcal{F}}\left\{\mathrm{f}_{\substack{\text{pulses}\\ \text{turbine } i}}(t-\mu_i)\right\}\right| = \left|\boldsymbol{\mathcal{F}}\left\{\mathrm{f}_{\substack{\text{pulse train centered}\\ \text{in time origin}}}(t)\right\}\right| \quad (31)$$

Therefore, the module of the Fourier transform of a pulse is independent of its position and can be easily calculated:

$$|\overrightarrow{c_m}|_{\substack{\text{single}\\ \text{turbine}}} = \frac{2}{T}\left|F_{\substack{\text{single}\\ \text{pulse}}}\left(\frac{2\pi m}{T}\right)\right| \quad (32)$$

- Since μ_i is uniformly distributed in $[0, T]$, the argument of $\boldsymbol{\mathcal{F}}\{\mathrm{f}_{\text{pulses}}(t-\mu_i)\}$ is also uniformly distributed in $[-\pi, +\pi]$. Thus, the angles $2\pi\mu_i/T$ and $2\pi\mu_k/T$ at which pulses i and k occur are independent random variables.

$$\mu_i \sim Uniform[0,\, T] \Rightarrow \frac{2\pi}{T}\mu_i \sim Uniform[0,\, 2\pi] \quad (33)$$

- If $m \neq 0$ and $m \in \mathbb{N}$, $e^{j2\pi m\mu_i/T}$ is a phasor of unity modulus and random argument. For the usual number of turbines in a wind farm ($N > 4$), the sum of these phasors, $\sum e^{j2\pi m\mu_i/T}$, is approximately a complex normal random variable with zero mean and standard deviation $\sqrt{N/2}$ (see [4]).

$$\sum_{i=1}^{N} e^{+j\frac{2\pi m}{T}\mu_i} \sim \mathbb{C}N(0, \sqrt{N/2}) \qquad m \neq 0, \quad m \in \mathbb{N} \quad (34)$$

- If $m \neq 0$, $\overrightarrow{c_m}$ is the sum of N phasors of random argument and fixed module.

$$\overrightarrow{c_m} = -\frac{2}{T} F_{\substack{\text{single}\\ \text{pulse}}}\left(\frac{2\pi m}{T}\right)\sum_{i=1}^{N} e^{+j\frac{2\pi m}{T}\mu_i} \quad (35)$$

- Summarizing, the complex Fourier coefficients $\overrightarrow{c_m}$ are, approximately, complex normal random variables with zero mean and standard deviation $\sigma_{\vec{c}_m}$.

$$\overrightarrow{c_m} \sim \mathbb{C}N(0, \sigma_{\vec{c}_m})$$

$$\sigma_{\vec{c}_m} = \frac{\sqrt{2N}}{T} F_{\substack{\text{single}\\ \text{pulse}}}\left(\frac{2\pi m}{T}\right) \qquad m \neq 0 \quad (37)$$

The argument of $\overrightarrow{c_m}$ is uniformly distributed in $[-\pi, +\pi]$ and the modulus $|\overrightarrow{c_m}|$ has a Rayleigh distribution with parameter $\sigma_{\vec{c}_m}$.

The zero order coefficient $\overrightarrow{c_0}$ or c_0 is twice the DC component of the signal (i.e., twice the average wind farm power).

$$\langle c_0 \rangle = 2\langle P_{farm} \rangle \quad (38)$$

2) Root Mean Square (RMS) value of the power fluctuations due to tower shadow and wind shear at the wind farm output

The Root Mean Square (RMS) value of the power fluctuations, RMS_{farm}, is a figure that characterizes the overall oscillation of a the wind farm output due to tower shadow and wind shear. It is the standard deviation of the sum of power pulses or the root of the squared sum of the modulus of Fourier coefficients. The calculus of RMS_{farm} from Fourier coefficients can be derived using Parseval's Theorem.

$$RMS_{\Delta Pfarm}{}^2 = \frac{1}{2}\sum_{m=1}^{\infty}|\overrightarrow{c_m}|^2 = \frac{1}{2}\sum_{m=1}^{\infty}\overrightarrow{c_m}\cdot\overrightarrow{c_m}^* \quad (39)$$

$$\langle RMS_{\Delta Pfarm}{}^2 \rangle = \frac{1}{2}\sum_{m=1}^{\infty}\langle \overrightarrow{c_m}\cdot\overrightarrow{c_m}^* \rangle = \frac{1}{2}\sum_{m=1}^{\infty} Cov[\overrightarrow{c_m}, \overrightarrow{c_m}] =$$

$$= \sum_{m=1}^{\infty}\sigma_m{}^2 = \sum_{m=1}^{\infty}\left[\frac{\sqrt{2N}}{T}F_{\text{pulse}}(2\pi m/T)\right]^2 \quad (40)$$

The RMS value of fluctuations can be also equivalently derived for any single pulse shape using the following relationship:

$$\langle RMS_{\Delta Pfarm}{}^2 \rangle = \quad (41)$$

$$= N\left[\frac{1}{T}\int_{-T/2}^{+T/2}\mathrm{f}_{\substack{\text{single}\\ \text{pulse}}}{}^2(t)\,dt - \left(\frac{1}{T}\int_{-T/2}^{+T/2}\mathrm{f}_{\substack{\text{single}\\ \text{pulse}}}(t)\,dt\right)^2\right]$$

Mean value of squared RMS_{farm} for rectangular, triangular and Gaussian shape of pulses as defined in (1) to (8) is:

$$\langle RMS_{\Delta Pfarm}{}^2 \rangle = N\left(\frac{\alpha\,\tau}{T}\right)^2\left(k\frac{T}{\tau} - 1\right) \quad (42)$$

Where the constant k depends on the pulse shape

$$k = \frac{T}{\alpha^2\,\tau}\int_{-T/2}^{+T/2}\mathrm{f}_{\substack{\text{single}\\ \text{pulse}}}{}^2(t)\,dt = \begin{cases} 1 & \text{rectangular pulse} \\ 2/3 & \text{triangular pulse} \\ 1/\sqrt{2} & \text{gaussian pulse} \end{cases} \quad (43)$$

The distribution of $RMS_{farm}{}^2$ can be derived from the distribution of wind farm power output.

$$CDF_{RMS^2\,\Delta Pfarm}(x) = CDF_{\Delta Pwind\ farm}(\sqrt{x} + N\alpha\,\tau/T) \quad (44)$$

where $CDF_{\Delta Pwind\ farm}(x)$ has been computed for rectangular in (17).

In farms with a many turbines, the distribution of farm output y due to tower shadow converges asymptotically to a normal distribution with mean $\langle y \rangle = N\alpha\,\tau / T$ and variance $Var(y) = \left\langle y^2 - \langle y \rangle^2 \right\rangle = \left\langle RMS_{\Delta Pfarm}{}^2 \right\rangle$. Thus, the modulus of the fluctuation, $RMS_{\Delta Pfarm}$, is distributed as a Rayleigh random variable with scale parameter equal to standard deviation of the underlying cyclic stochastic process [6].

$$\underset{N\to\infty}{Lim}\, RMS_{\Delta Pfarm} \rightarrow Rayleigh\left(\sqrt{\left\langle RMS_{\Delta Pfarm}{}^2 \right\rangle}\right) \tag{45}$$

The average of RMS_{farm} is a bit smaller than the squared root of the $RMS_{farm}{}^2$ mean since quadratic averages weigh up larger values. (46)

$$\left\langle RMS_{\Delta Pfarm} \right\rangle \approx \sqrt{\frac{\pi}{2}}\sqrt{\left\langle RMS_{\Delta Pfarm}{}^2 \right\rangle} < \sqrt{\left\langle RMS_{\Delta Pfarm}{}^2 \right\rangle}$$

Mean RMS_{farm} for rectangular, triangular and Gaussian pulses is shown bellow.

$$\left\langle RMS_{\Delta Pfarm} \right\rangle \approx \frac{\alpha\,\tau}{T}\sqrt{N}\sqrt{\frac{\pi}{2}}\sqrt{k\frac{T}{\tau} - 1} \tag{47}$$

Since usually $1 \ll T / \tau \approx 6 \sim 10$, the fluctuation is also proportional to the square root of the relative width of the pulse $\sqrt{\tau / T}$, approximately.

$$\left\langle RMS_{\Delta Pfarm} \right\rangle \underset{approx}{\simeq} \alpha\sqrt{N}\sqrt{\frac{\tau}{T}}\sqrt{\frac{k\pi}{2}} \tag{48}$$

Notice that $RMS_{\Delta Pfarm}$ is proportional to the pulse power dip amplitude α and to the square root of the number of turbines $\sqrt{N}$ and the relative width of the pulse, $\sqrt{\tau/T}$.

3) Distribution of the gradient of power (time derivative of power) due to tower shadow effect

The derivative of farm power is a measure of the variability of farm output with time. Whereas RMS_{farm} only account the deviation from the average of the power output, the distribution of time gradient of power dP_{farm} / dt measures how quickly are the oscillations due to the position of the turbine blades.

The distribution of the gradient of power (time derivative of power) can be computed using properties of Fourier transforms in a similar fashion to the previous section.

$$\frac{dP_{farm}}{dt} = \mathrm{Re}\left(\sum_{m=1}^{\infty} j\frac{2\pi m}{T}\,\overrightarrow{c_m}\, e^{j\frac{2\pi m}{T}t}\right) \tag{49}$$

$$\left\langle \frac{dP_{farm}}{dt}^2 \right\rangle = \frac{1}{2}\sum_{m=1}^{\infty}\left(\frac{2\pi m}{T}\right)^2 \left\langle \overrightarrow{c_m}\cdot\overrightarrow{c_m}^* \right\rangle =$$

$$= \frac{1}{2}\sum_{m=1}^{\infty}\left(\frac{2\pi m}{T}\right)^2 Cov[\overrightarrow{c_m}, \overrightarrow{c_m}] = \sum_{m=1}^{\infty}\left(\frac{2\pi m}{T}\sigma_m\right)^2$$

The sum can be transformed using (37):

$$\left\langle \frac{dP_{farm}}{dt}^2 \right\rangle = \sum_{m=1}^{\infty}\left[\frac{2\pi m}{T}\frac{\sqrt{2N}}{T}F_{\mathrm{pulse}}\left(\frac{2\pi m}{T}\right)\right]^2 \tag{50}$$

Notice that square pulses presents infinite derivative at flanks and its quadratic average is infinite (real pulse are continuous functions). For triangular and Gaussian pulses, the RMS value of the time derivative is:

$$\sqrt{\left\langle \frac{dP_{farm}}{dt}^2 \right\rangle} = \begin{cases} \alpha\sqrt{2\dfrac{N}{\tau\cdot T}} & triangular\ shape \\ \alpha\sqrt{\dfrac{\pi}{\sqrt{2}}\dfrac{N}{\tau\cdot T}} & gaussian\ shape \end{cases} \tag{51}$$

The RMS value of the derivative of farm power is proportional to turbine pulse height α and to the square root of the number of turbines $\sqrt{N}$. Notice that the time derivative of farm power is inversely proportional to *square root* of the product of turbine time constants. If the turbine pulse is symmetric respect its peak, as all the pulses presented in this work, the distribution of power gradient is also symmetric respect to zero level.

III. CONCLUSIONS

The power fluctuation when a blade passes in front of the tower can be represented by a power dip event. The number of simultaneous tower shadows happening at an instant is a binomial random variable.

Tower shadow events are characterized as a generalized Poisson process and its average frequency is N times the tower shadow frequency of a single turbine. Notwithstanding this fact, the standard deviation of the wind farm power output due to tower shadow and its effective gradient are proportional to $\sqrt{N}$.

The tower shadows at a wind farm can be eventually compensated if the turbines could be controlled to distribute their rotor angle evenly. However this is not practical because this control would impose the same rotor speed at all turbines.

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