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# Wind Power Variability Model
## Part I – Foundations

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***Abstract*—A novel technique to account wind variability is presented based on Markov Chains and classification of observations. This model describes the power system status through combination of cases or "snapshots of the network" obtained from the clustering of observations.**

**The occurrence and variability of each case is modeled by a Markov Chain. This approach models the non-linear conventional behavior of the farms but also events that rarely happens but that they have a high impact in system reliability and stability (such as sudden disconnection of generators due to grid perturbations, swift change in wind during storms, etc).**

**This model requires running just as many power flows as states has the system and it allows to derive easily and rigorously the probability of events. Moreover, the regulation of spinning reserve or reactive control can be easily optimized using Markov Decision Processes.**

**The use of network snapshots allows to use a full model of the grid (instead of linear models). Intermediate cases are interpolated using fuzzy clustering, reducing the required number of states for a given accuracy.**

**To explain adequately the foundations and to show the potential applications of this approach, this work has been divided in three parts. In this part, the theoretical foundations and an overview of the method are presented. The second part shows the estimation of Markov Parameters for a system with three wind farms. The third part illustrates the stochastic power flow of the three wind farms and introduces the possible optimization through Markov Decision Processes.**

***Index Terms*— wind power, Markov chain, variability, power quality, spinning reserve.**

## NOMENCLATURE

$a_{ij}[k]$ = probability of having observed a transition from state $i$ to $j$ at instant $k$.
$\beta$ = confidence level
$F_i$ = observed occurrences of state $i$
$F_{ij}$ = observed transitions from state $i$ to $j$
$m$=number of states of the Markov chain.
$\mathbf{P} = [p_{ij}]$ = transition matrix of the Markov chain.
$p_{ij}$ = forward transition probabilities from state $i$ to state $j$ of the Markov chain.
$\hat{\mathbf{P}} = [\hat{p}_{ij}]$ = estimate of transition probability of the Markov chain.
$\tilde{\mathbf{P}} = [\tilde{p}_{ij}]$ = backward transition probabilities.
$\boldsymbol{\pi} = [\pi_1, \pi_2, \ldots, \pi_m]$ = the stationary distribution of the Markov chain.
$s$ = number of wind farms whose power output is measured.
$\mathbf{x}[k]=[x_1[k],\ x_2[k],\ \ldots,\ x_s[k]]$ = vector of wind farm power output at instant $k$.
$u[k]$= most likely state for the observation $\mathbf{x}[k]$
$\mathbf{v}[k]=[v_1[k],\ v_2[k],\ \ldots,\ v_s[k]]$ = vector of wind farm power output in the canonical basis of left eigenvectors of $\hat{\mathbf{P}}$ at instant $k$.
$\mathbf{z}[k]=[z_1[\mathbf{x}[k]],\ z_2[\mathbf{x}[k]],\ \ldots,\ z_m[\mathbf{x}[k]]]$ = output of the fuzzy classification representing the state probability at instant $k$.

## I. INTRODUCTION

WIND speed fluctuations are usually analyzed through linear mathematical tools such as frequency spectrum and time series. The Van der Hoven's wind spectra [1] show a gap between 3 minutes/cycle and 5 hours/cycle that separates fast fluctuations from slow fluctuations. Nevertheless, this division is not so clear at some locations [2, 3, 4].

On the one hand, slow fluctuations are mainly due to meteorological dynamics and they are widely correlated spatially and temporally. Slow fluctuations in power output of near farms are quite correlated and wind forecast models try to predict them to optimize power dispatch. On the other hand, fast wind speed fluctuations are mainly due to turbulence and microsite dynamics [5].

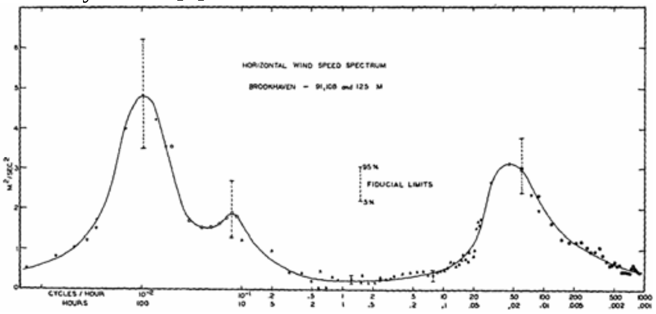

Fig. 1: Van der Hoven's spectral model (from [1]).

## II. Step Changes in Power Output

Wind turbines can cause a periodic behavior if they experience repetitive connection and disconnections due to difficult operating conditions (wind speed near cut-in or cut-out, high temperatures, high turbulence, etc.). In Fig. 2, the active power output of a single turbine has extreme variations due to a combination of high ambient temperature and high wind, yielding to high temperature alarms at gearbox oil. In that situation, the power output of the farm is not so abrupt because even though this behavior was common to many turbines, the disconnection and connections of the turbines were not synchronized.

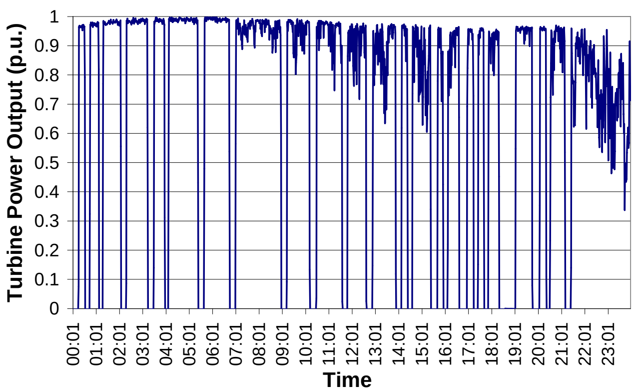


Fig. 2: Power output of a single turbine experiencing 24 repetitive stops due to over temperature in 20/07/1998 (24 h).

The repetitive connection and disconnection of up to two turbines is a reduced portion of the total active wind farm power output expressed in p.u. (see Fig. 3).

The sudden wind change can also cause variations in power output of the farm in minutes, as can be seen at Fig. 4. At 17:25, the power output of a farm was 0.21 p.u. and ten minutes later was 0.96 p.u. due to a storm.

In general, power variations as extreme as Fig. 4 are smoothed in the total generation of a bigger area. However, even in a wide area such as Spain with 8000 MW of wind power installed, a variation rate of 1000 MW/hour approximately can be seen in Fig. 5 [6].

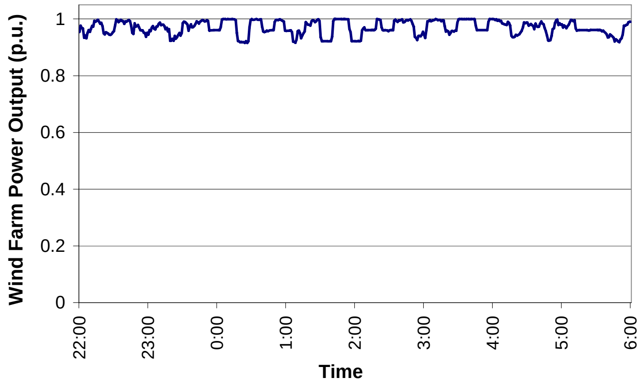


Fig. 3: Active power output of a wind farm with 26 turbines experiencing repetitive connection and disconnection of up to two turbines due to internal errors in 7/02/1999 (mean speed at meteorological mast was around 14 m/s).

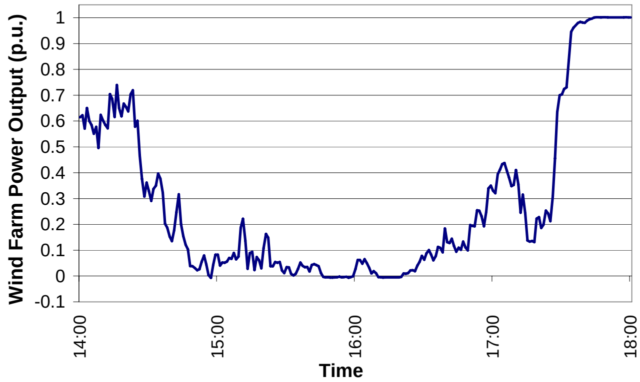


Fig. 4: Active power output of a wind farm experiencing high variability in 9/02/1999 due to a sudden change in the weather at 17:30.

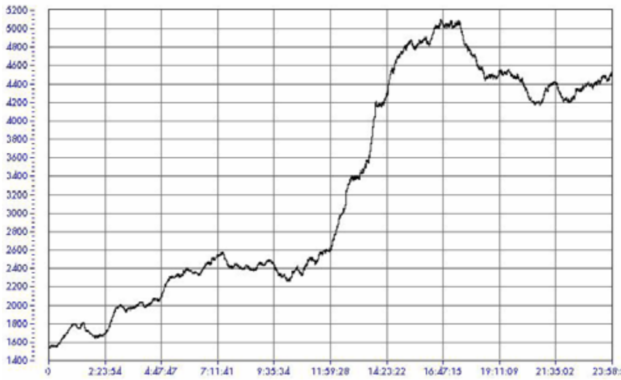

Fig. 5: Active power output (MW) of Spanish wind farms experiencing high increase in 18/01/2005 between 12:00 and 15:00.

The worse case is when the circuit breaker disconnects a wind farm. The Spanish Ministerial Order of 5-9-1985 [7] ordered that the protection relays of wind substations were adjusted very strictly (for example, instantaneous trip for voltages under 0,85 p.u. or over 1,1 p.u.). This caused a number of unjustified disconnection of wind farms at network contingencies. In Fig. 6, recovering normal production from wind farm energization lasted three minutes (with Vestas Opti-Slip 600 kW turbines). Nowadays, the relays are adjusted more selectively and the turbines are rewarded for fault riding capabilities (even though [7] hasn't formally repealed, up to now).

To sum up, some events in the wind farm produce step changes in the output and they are very difficult to model using frequency or time series analysis.

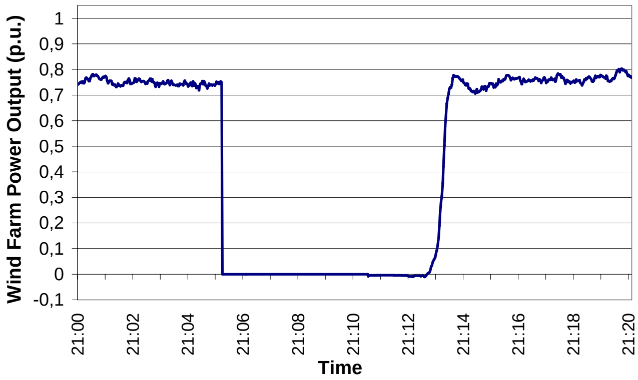


Fig. 6: Active power output of a wind farm experiencing a disconnection in 3/02/1999 due to a trip of the homopolar protection relay between 21:05:24 and 21:10:55. Three minutes later, the output reached normal values.

## III. Time-Averaged vs Instantaneous Values

Most SCADA, data loggers and energy metering devices record average power and other variables in five to fifteen minute intervals. The standard time interval is 10 minutes for power curves and flicker [8, 9] and 15 minutes for reactive power billing [10] (a suitable integrating period for both task is 5 minutes and its integer fractions).

Due to lack of detailed data, the change rate between two consecutive time intervals would be computed with (1) from recorded data.

$$\frac{dP(t)}{dt} \approx \frac{\Delta P(kT_{\log})}{\Delta t} = \frac{P(kT_{\log}) - P((k-1)T_{\log})}{T_{\log}} \qquad (1)$$

Change rate is equal to (1) only if change rate is constant –in other cases, (1) would be the mean rate of change–. That is a reasonable approximation during continuous operation.

However, data from power loggers are ambiguous at switching events. According to the Nyquist–Shannon sampling theorem, the highest frequency that can be reconstructed with measures logged each time interval $T_{log}$ is $f_{Nyquist} = 1/(2\cdot T_{log})$. In other words, the maximum frequency of fluctuations that can be analyzed is half of the logging frequency.

The moving average can be regarded as the convolution of the instantaneous power of the wind farm with a pulse of width $T_{log}$ (see Fig. 8). Afterwards, the moving average is sampled with $T_{log}$ period. The continuous transfer function of the average power in an interval $T_{log}$ is shown in Fig. 8, up to the logging frequency of the power analyzer, $f_{\log} = 1 / T_{\log}$ :

$$|F(f)| = \left( \frac{\sin\left(\pi f T_{\log}\right)}{\pi f T_{\log}} \right)^2 \qquad (2)$$

where $|F(f)|$ is the frequency response (i.e. the attenuation of frequencies due to discrete logging of parameters).

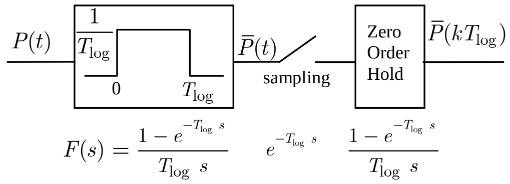


Fig. 7: Model of the wattmeter of the wind farm.

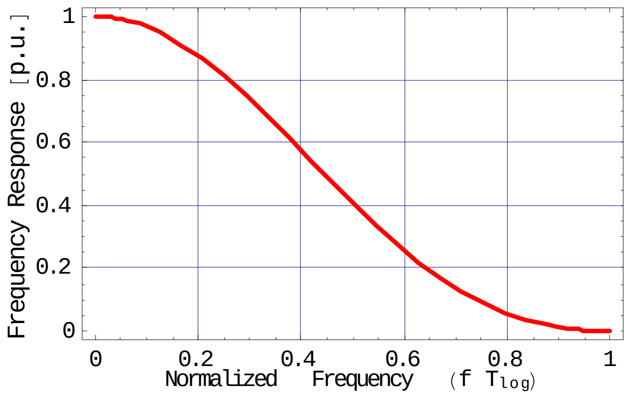


Fig. 8: Plot of frequency response of the network analyzer.

## IV. Statistical Approach To Variability

Linear stochastic tools such as time series or frequency analysis are very popular for characterizing the farm output despite casual individual turbine disconnections. However, Markov chains will be used in this work since:

- The behavior at low or high wind is very different from middle wind. Such behavior is highly non-linear.
- Some stochastic models do not model adequately that power output is constrained from zero to full generation (0 to 1 p.u.). The long run probability of the power output is bimodal, showing more steady operation at full generation or at no generation [11]. The output can vary suddenly from 0 to 100% in wind park switching events.
- Grid disturbances can trip a great amount of wind power, which can be hardly characterized with stochastic linear models based on time series or frequency analysis. Big fluctuations such as disconnection of a whole park or a group of farms are not suited for spectrum neither time series analysis because abrupt changes involve high components in all frequencies. In contrast, the probability of a sudden change can be modeled easily with Markov Chains.
- The linearized model of an electrical system is not adequate for severe perturbations.

This work will focus in a non-linear stochastic characterization of power output through a finite number of states.

Markov chains have been chosen for this work due to its simple mathematical treatment and its superior theoretical properties for stochastic dynamics. This model is also well suited for stochastic power flows and for understanding system dynamics.

A stochastic process can be modeled by a Markov model if the evolution of the system is only dependent of the present.

In other words, a Markov model implies considering the process memoryless. The utilization of regular Markov Chains imply that the permanence of the system in a state is distributed statistically exponentially (or geometrically in the case time is discretized). To override the memoryless characteristic, the tendency of the system during the last hours or the wind forecast for a given horizon can be included as another parameter of the states (at the cost of a bigger number of states).

## V. Characterization of Power Variability of Wind Generation with Markov Chains

Markov chains have been used in modeling physical, biological, social, and engineering system such as population dynamics, queuing networks and manufacturing systems. One of the main advantages of using Markovian models is that they are general enough to capture the dominant factors of system uncertainty and, in the meantime, it is mathematically tractable.

Most dynamic systems in the real world such as meteorology are inevitably large and complex, mainly due to their interactions with numerous subsystems. Since exact or closed-form solutions to such large systems are difficult to obtain and they would require extensive measures, one often has to be contented with approximate solutions. Take the optimal control of a dynamic system such as spinning reserve in a power system due to wind power. Because the precise mathematical models are difficult to establish, near-optimal controls often become a viable, and sometimes the only alternative. Such near optimality requires much less computational effort and often results in more robust policy to attenuate unwanted disturbances [12].

Wind power show different prevailing dynamics when it is analyzed for a few milliseconds or for a daily horizon. It can be thought that electromechanical dynamics used different time scale from the weather evolution.

The division between fast and slow dynamics makes easier large-scale optimization of wind energy. If all the important factors are included in a Markov Model, it would lead to a large state space with many parameters to estimate and an exhaustive and extensive measuring system. To reduce the complexity, a hierarchical approach is suggested, which leads to a multi-resolution formulation. The hierarchical approach relies on decomposing the states of the Markov chains (all the possible combination of power output of wind farms) into several recurrent classes (typical patterns of generation observed in power output of wind farms). The essence is that within each recurrent class the interactions are strong and among different recurrent classes the interactions are weak.

Traditionally, Markov chains have been applied in Electrical Engineering for the study of queues [13] and power system reliability given rate of failure and reposition times of its components. In Markov Chain Monte Carlo (MCMC) simulations, Markov Chains are employed as random number generators with particular characteristics [14], not in the way they are utilized in this paper.

### *A. State selection*

In this proposed methodology, each Markov state can be seen as a case that characterizes a typical operational mode of the wind farm (or a group of wind farms). Full generation, no generation and partial generation are candidates for Markov states. If partial operation near cut-in wind speed is notably different from partial operation near rated wind speed, they should be considered as distinct Markov states. Fig. 9 shows this discretization and the arrows indicate a transition from a state to another one.

Fig. 9 is *a priori* arrangement, but the election of states can be optimized using a clustering algorithm which minimizes the classifying error and selects the optimum number of states [15, 16]. This is crucial when classifying data from several wind farms. Therefore, the clustering is used as a mathematical tool to transform a continuous multivariate space $\mathbb{R}^s$ (the active power output of $s$ wind farms) into a discrete and finite (numbered in $\mathbb{N}$) state space to use Discrete Time Markov Chains with convenient matrix algebra instead of functional analysis.

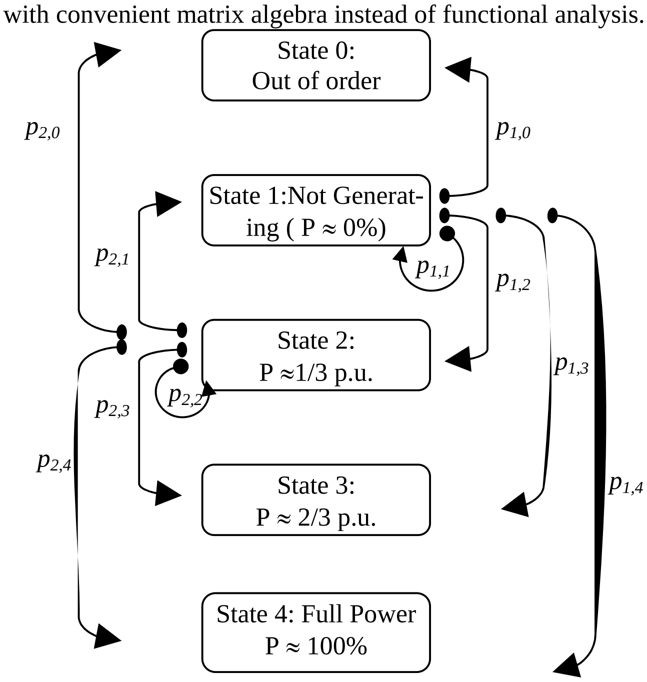


Fig. 9: Discretization of power output of one wind farm into a number of states (four in this figure). Only transitions from states 1 and 2 are shown for clarity.

The use of different states allows to use a full model of the grid (instead of the classical small-signal model) and the state weighting describe intermediate cases reducing the required number of states, $m$. The combination of matrix algebra and state probability imply the (linear) interpolation between the centroids of the states for describing intermediate cases.

### *B. Estimation of* $\mathbf{P}$ *from conventional clustering.*

Time Markov chains are a powerful tool to cope with states and transitions between states. The dynamics of a system such as a wind farm or a group of them are characterized by a transition matrix $\mathbf{P} = [\, p_{i,j} \,]$. The dynamics of the system are characterized through the transition probabilities from state $i$ to state $j$, $p_{i,j}$, which are estimated from actual data. The probability of staying in the same state the next interval is $p_{i,i}$. The residence time in a state (time during the system is at state *i*) is distributed exponentially and its characteristic value can be derived from transition matrix.

$\hat{\mathbf{P}}$ is an estimate of the transition matrix $\mathbf{P}$. If the output of the classification algorithm for each sample $k$ is just the state number $u[k]$, one can find the transition occurrence $F_{ij}$ in the sequence by counting the number of transitions from state $i$ to state $j$ in one step.

$$\mathbf{F} = \begin{bmatrix} \text{matrix of} \\ \text{observed transitions} \end{bmatrix} = \begin{bmatrix} F_{11} & \cdots & \cdots & F_{1m} \\ F_{21} & \cdots & \cdots & F_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ F_{m1} & \cdots & \cdots & F_{mm} \end{bmatrix} \quad (3)$$

and $F_i = \text{observed occurrences of state } i = \sum_{k=1}^{m} F_{ki}$ (4)

Then the one-step transition matrix $\mathbf{P}$ can be estimate as follows:

$$\hat{\mathbf{P}} = \text{estimate of } \mathbf{P} = \begin{bmatrix} \hat{p}_{11} & \cdots & \cdots & \hat{p}_{1m} \\ \hat{p}_{21} & \cdots & \cdots & \hat{p}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{p}_{m1} & \cdots & \cdots & \hat{p}_{mm} \end{bmatrix} \quad (5)$$

The elements of $\hat{\mathbf{P}} = [\hat{p}_{ij}]$ can be computed according to (6) (from 6.4.2 in [17])

$$\hat{p}_{ij} = \frac{\text{observed transitions from state } i \text{ to } j}{\text{ocurrences of state } i} = \frac{F_{ij}}{F_i} \quad (6)$$

Since states represent typical operational conditions, all states eventually occur in the sample set ($F_i > 0$).

For observed transitions ($F_{ij} > 0$), the standard error of $\hat{p}_{ij}$ is approximately:

$$\widehat{se} = \sqrt{\frac{\hat{p}_{ij}\left(1 - \hat{p}_{ij}\right)}{F_i}} \quad (7)$$

The unobserved transitions ($F_{ij} = 0$) can be due to real unfeasibility or to the limited available data. If $F_{ij} = 0$, the transition probability $\hat{p}_{ij}$ is bound to $[0, 1-(1-\beta)^{1/F_i})$ with confidence level $\beta$ ($F_{ij}$ is binomially distributed). In part II of this paper, the estimation of rare events will be revised.

### *C. Improving state estimation*

A further refinement is to quantify the similarity of real data to each cluster. For example, fuzzy classification computes the similarity of each observation with each state and the fuzzy membership degree can be interpreted as the probabilities that the measures corresponds to the states of the Markov model [18]. This approach improves the performance of the Markov Model since wind farm output is continuous and there is not a definite division or separation between clusters.

There are powerful clustering algorithms where any real observation is classified into a group with an error that can be controlled. Since Markov Chains poses a probabilistic discretization into states and the cluster boundary is blur in wind characterization[19], fuzzy c-means clustering is a suitable clustering algorithm. In fuzzy clustering, each datum has a degree of belonging to clusters, as in fuzzy logic, rather than belonging completely to just one cluster [20]. Thus, data on the boundary of a cluster may be in the cluster to a lesser degree than points near the centroid. For each observation of the $s$ wind farm outputs $\mathbf{x}[k]=[x_1[k],\ x_2[k],\ \ldots,\ x_s[k]]$ we have a coefficient $z_i(\mathbf{x}[k])$ giving the degree of membership to the $i^{\text{th}}$ cluster ($1 \le i \le m$). Usually, the sum of those coefficients is defined to be 1, $\Sigma_{i=1}^{m} z_i\left(\mathbf{x}[k]\right) = 1$, so that $z_i(\mathbf{x}[k])$ denotes a probability of belonging to cluster $i$ and $\mathbf{z}[k]=[z_1[\mathbf{x}[k]],\ z_2[\mathbf{x}[k]],\ \ldots,\ z_m[\mathbf{x}[k]]]$ is the probability vector. The obtained clusters implicitly model the relationship among random variables whereas its probability is computed from occurrence frequency of real data.

If operation of a farm or a cluster of farms are significantly influenced by other factors, those features can be included in the classification process but at the cost of increasing the data requirements. Therefore, each state can be classified mainly by its power output and secondary, by other parameters such as average wind direction, meteorological stability or wind prediction for a given horizon.

At a given time, the best classification of a big system into a reduced set of states can be challenging. The best procedure for estimating the system state depends of available data, aim of the analysis and philosophical implications of each approach.

Consider the following example: near cut-off wind speeds, some turbines are stopped whereas others remain at full power. The overall situation will correspond to full generation in a portion and no generation in the rest. That intermediate situation can be represented by the probability of pertaining to full and no generation states (if the number of generating and installed turbines are known, the probability of full generation status can be interpreted as their ratio –frequentist interpretation of probability–). Using the Bayesian interpretation of statistics, the probability of full or no generation is the degree of belief that the real situation corresponds to each state. Using the interpretation of fuzzy logic, the possibility of each state is the membership grade to a fuzzy set. From the mathematical theory of approximating functions, the state probability can be regarded as interpolating coefficients used to represent piecewise functions such as expected values and voltage or power flow profiles at some nodes. In Hidden Markov Processes, the real state is not observable usually but it can be inferred from observations trough Viterbi algorithm and the emission matrix which relates measured parameters and unobservable states.

### *D. Estimation of* $\mathbf{P}$ *from fuzzy clustering.*

On one level, the average power at an interval can be near the classification boundary of two states and the instantaneous state could be considered as a partially corresponding to adjacent states. The instantaneous state can vary inside the time interval and the output of the classification process will be stochastic. If the classification of two consecutive periods is the same, the system would be regarded as "continuing" in the same state (although the actual process is more complex). On another level, if the states are very similar a fuzzy classification is required to avoid overestimating transitions due to sharp cluster boundaries.

The classification algorithm is the level of membership to each state $z_i[k] = [z_1(\mathbf{x}[\mathrm{k}]), z_2(\mathbf{x}[\mathrm{k}]), \ldots, z_k(\mathbf{x}[\mathrm{k}])]$, measured in observation $k$ (equivalent to the Bayesian interpretation of probability of the occurrence of state $i$ at observation $k$). The output of the classification algorithm is normalized, therefore:

$$\sum_{i=1}^{m} z_i[k] = 1 \tag{8}$$

where $m$ is the number of states in the system.

In fuzzy classification, the estimation of transitions from state *i* to *j* depend on some assumptions taken. Since wind is a meteorological spatial property which experience gradual changes, it is sensible to select the estimation that minimizes state variability. With this assumption, the $\hat{\mathbf{P}}$ can be estimated as follows.

$a_{ij}[k]$ = probability of having observed a transition from state $i$ to $j$ at instant $k$.

First, estimate the probability of continuing in the same state at instant $k$:

$$a_{ii}[k] = \min\left(z_i[k], z_i[k+1]\right) \tag{9}$$

Then, the probability of jumping to a different state at instant $k$ is estimated as the ratio of state probability variation in contiguous instants to the probability of not continuing in the previous state:

$$a_{ij}[k] = \begin{cases} 0 & \text{if} \quad \sum_{i=1}^{m} a_{ii}[k] = 1 \\ else & \dfrac{\left(z_j[k+1] - a_{jj}[k]\right)\left(z_i[k] - a_{ii}[k]\right)}{1 - \sum_{i=1}^{m} a_{ii}[k]} \end{cases} \tag{10}$$

Finally, estimate $\hat{\mathbf{P}} = [\hat{p}_{ij}]$ from observations:

$$F_{ij} = \sum_{k=1}^{N-1} a_{ij}[k]; \qquad F_i = \sum_{k=1}^{m} F_{ki}; \qquad \hat{p}_{ij} = \frac{F_{ij}}{F_i} \tag{11}$$

If the states are not directly observable, the Viterbi algorithm can be modified to estimate the most probable hidden state based on the fuzzy classification.

### *E. Considerations on* $\hat{\mathbf{P}}$.

Since the probability of state transitions is estimated from real data, this approach can handle abrupt behavior of the farms along with events that rarely happens but that they have a high impact in system reliability and stability (sudden disconnection of generators due to grid perturbations, swift change in wind during storms, etc).

The obtained Markov Chain is irreducible because starting from any state $i$, it is possible to enter state $j$ in finite number of transitions. This property will be assumed in the following sections. Moreover, if all transitions $\hat{p}_{ij}$ have non-null probability, the Markov chain is said also regular.

## VI. CONCLUSION

In this first part of the article, the theoretical foundations and an overview of a method to quantify uncontrolled generation variability have been outlined. Moreover, this model is well suited for probabilistic power flows and for stochastic optimal power flow through Markov Decision Process. The method is further explained and put into practice in parts II and III.

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