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Wind Power Variability Model
Part II – Probabilistic Power Flow

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***Abstract*— The classification of states can be based on power output, "unperturbed wind speed" and wind speed prediction, depending on available data and aim of the wind farm model. The performance matrix in Standard IEC 61400-12-3 can be used as emission matrix to relate wind and power in a wind farm using a Hidden Markov Model. The wind farm model can be used also as time interpolation between horizons of wind prediction or to account switching events such as sudden disconnection of the farm. The basic workflow to compute a stochastic power flow based in Markov Model is presented. A simplified, steady state, quadratic model of the wind farm is shown for justifying the approximation of networks to PQ nodes and the interpolation between states. This quadratic model can be used also to estimate the reactive power for steady state.**

I. Markov Model Based on Wind Parameters

The consideration of wind speed and direction along with wind power output can give further insight in wind farm dynamics than using only power output. However, it is usual to have only limited data (only wind parameters or only power generation).

Power Flows require the active and reactive power of all generation and consumption nodes. This subsection discusses the modifications needed to use a Markov Model based on wind and power parameters or only wind characteristics.

If the aim of the Markov model is to work with wind forecast, the state number can be defined based on mean wind speed and direction at the wind farm. The power output can be derived from the conditional probability of power output given wind speed and direction.

Standard IEC 61400-12-3 [1] shows a detailed method to compute the wind farm power output from "unperturbed wind speed" of the wind farm. The wind farm power curve consists of performance matrix **M** indicating the expected power output from wind speed and wind direction values. In Markov jargon, the state space can be built from wind speed and direction. The emission probability matrix can be the performance matrix **M** if the bins of IEC 61400-12-3 are elected as Markov states.

Moreover, Hidden Markov Models (HMM) can cope with more complex dynamics when system state is not directly observable (for example, if important information like turbine malfunctions and maintenance work are not available).

A model should be simple enough to avoid over-fitting or over-fluctuations. Even more, the use of very complex models with many parameters need big amounts of data to be adjusted and its interpretation becomes tougher.

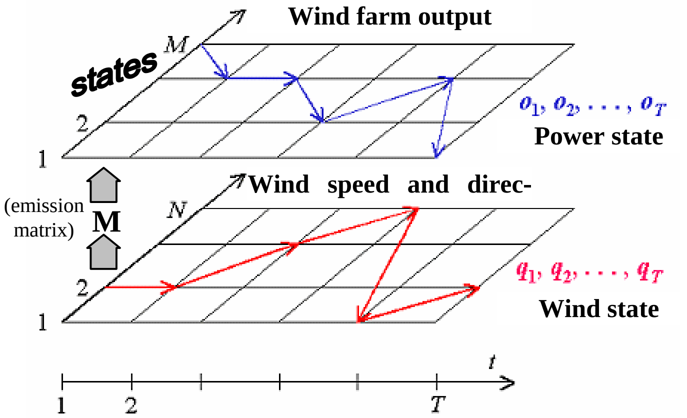

Fig. 1: Schematic relationship between measures (observations) and estimated states when they can not be derived straightforward from measures (from [2]).

II. Improving Markov Model with Weather Forecast

Weather forecast is a widespread tool to characterize wind farm power trend from 6 hours ahead. Meteorological physical models are much more precise for assessing power evolution for long horizons, whereas Markov Chains are more adequate for assessing power variability for short and medium horizons and for optimize network policy. Therefore, a model that combines weather forecast and a Markov Model is more suitable than just trying to use a more complex statistical distribution (a semi-Markov model has fewer theoretical properties, it increases model complexity and it does not account the complex weather dynamics).

The influence of meteorological dynamics can be incorporated using the weather forecast as another parameter. If weather forecast is not available, a Hidden Markov Chain (HMC) can be used for accounting the meteorological stability (an unobservable parameter). Other approach is the inclusion of the weather forecast and the farm availability in the classification process.

To sum up, if the time span of the estimation is bigger than 6 hours, weather forecast must be used to increase the accuracy of the wind power variability model.

III. Discerning Switching Events from Continuous Operation

Switching events are difficult to detect if there are no meteorological data available. Switching events can be guessed during high wind with a statistical hypothesis test based on maximum change of wind. If power has changed above the confidence level for the previous power output, the operation is not continuous up to a significance level.

Moreover, if the variability of power output during normal operation is characterized through a Hidden Markov Model, the Viterbi algorithm can be adapted to estimate the most likely sequence of states from measures.

The effect of switching events range from voltage variations to frequency drifts in smalls systems or tie line overloads in big systems [3]. For example, the sudden disconnection of big amounts of wind power in Spain can overload tie lines with France [4]. The sudden disconnection of such big amounts can be due only to severe network disturbances that are spread along the grid. These events are unpredictable.

The disconnection due to extreme weather is more gradual because of geographical diversity of turbines. Some wind farms will experience greater wind speeds. Inside a wind farm, the more exposed turbines would shut down first. Moreover, extreme wind can be forecasted with some accuracy and the spinning reserve can be appropriately increased [5, 6]. Sometimes, the maximum and minimum power in the interval is also measured. This extra information is very valuable to discern switching events from very fast changes in wind.

IV. Stochastic Time Interpolation

Sometimes, wind or power forecast is given only at some time horizons and the power output should be computed at intermediate instants between actual data and forecasted value. Time interpolation can be performed using maximum a posteriori probability criterion according to a Markov Model.

The sequence of power output or wind speed and direction analyzed “as time goes by” is called the forward Markov process and it has a probability transition matrix usually denoted by $\mathbf{P}$ and its elements p_{ij}, the transition probabilities from state *i* to state *j*. However, the sequence ordered in reverse time direction is another Markov process [7] with backward transition probability matrix $\tilde{\mathbf{P}}$ and its elements, $\tilde{p}_{ij}$ the reverse transition probabilities from subsequent state *i* to the preceding state *j*:

$$\tilde{p}_{ij} = \pi_j \; p_{ji} \; / \, \pi_i \tag{1}$$

where π_i is the stationary distribution of the models and can be computed as the eigenvector for the unity eigenvalue of matrix $\mathbf{P}$ (or $\tilde{\mathbf{P}}$) or alternatively, as any row of the limiting distribution for long time horizons, $\lim_{N\to\infty} \mathbf{P}^N$.

A. Input data

$\mathbf{x}[0] = [x_1[0],\ x_2[0],\ \ldots,\ x_m[0]]$ = row vector of initial probabilities of all m states.

$\mathbf{x}_{forecasted}[N] = [x_1[N],\ x_2[N],\ \ldots,\ x_m[N]]$ = forecasted probabilities of all states for time horizon N.

$$\begin{aligned} \hat{\mathbf{x}}[N] &= \text{Expected value of } \mathbf{x}[N] = \mathrm{E}(\mathbf{x}[N]) = \\ &= \text{Probability}(\mathrm{X}[N] \neq \mathbf{x}_{forecasted}[N]) \cdot \mathbf{x}[0] \cdot \mathbf{P}^N + \\ &+ \text{Probability}(\mathrm{X}[N] = \mathbf{x}_{forecasted}[N]) \cdot \mathbf{x}_{forecasted}[N] = \\ &= (1-\beta)\, \mathbf{x}[0]\, \mathbf{P}^N + \beta\, \mathbf{x}_{forecasted}[N] \end{aligned} \tag{2}$$

β = confidence level of forecasted value $\mathbf{x}_{forecasted}[N]$

B. Estimation of the sequence of states

$$\begin{aligned} \hat{\mathbf{x}}[k] &= \text{Expected value of } \mathbf{x}[k],\ 0 \le k \le N = \mathrm{E}(\mathbf{x}[k]) = \\ &= \text{Probability}(\mathrm{X}[k] = \mathbf{x}[0] \cdot \mathbf{P}^k) \cdot x[0] \cdot \mathbf{P}^k + \\ &+ \text{Probability}(\mathrm{X}[k] = \mathbf{x}[N] \cdot \tilde{\mathbf{P}}^{N-k}) \cdot \mathbf{x}[N] \cdot \tilde{\mathbf{P}}^{N-k} \end{aligned} \tag{3}$$

The expected system evolution is the weighted sum of a forward and a backward Markov Process. In absence of relevant information, the weighting of the forward and backward process at point k can be proportional to the distance to the initial and end of the time interval. Accordingly, the following formulae expressing the probability of each state ($\tilde{\mathbf{P}}$ is supposed invertible in wind power applications):

$$\begin{aligned} \hat{\mathbf{x}}[k] &= \mathrm{E}(\mathbf{x}[k]) = \frac{(N-k)\, \mathbf{x}[0]\, \mathbf{P}^k + k\, \hat{\mathbf{x}}[N]\, \tilde{\mathbf{P}}^{N-k}}{N} = \\ &= \left(1 - \beta \frac{k}{N}\right) \mathbf{x}[0]\, \mathbf{P}^k + \beta \frac{k}{N} \mathbf{x}_{forecasted}[N]\, \tilde{\mathbf{P}}^{N-k} = \\ &= \left(1 - k \frac{\beta}{N}\right) \mathbf{x}[0]\, \mathbf{P}^k + k \frac{\beta}{N} \mathbf{x}_{forecasted}[N]\, \tilde{\mathbf{P}}^N \left(\tilde{\mathbf{P}}^{-1}\right)^k \end{aligned} \tag{4}$$

The latter formula can be expressed more compact as:

$$\hat{\mathbf{x}}[k] = \left[\mathbf{x}[0] + k \frac{\beta}{N} \left(\mathbf{x}_{forecasted}[N]\, \tilde{\mathbf{P}}^N - \mathbf{x}[0]\right)\right] \mathbf{P}^k \tag{5}$$

If the interpolation between the measured and the forecasted values is done geometrically instead of arithmetically, a similar formula can be derived.

The probability of any sequence of generation states can be computed as in chapter 7 of [8] (for example, probability of the sequence: full generation to no generation and then to full generation in successive time intervals).

V. Stochastic Power Flows

Probabilistic power flow is a term that refers to power flow analysis methods that directly treat the uncertainty of electric load, generation and grid parameters.

Classical approaches usually rely on simplifications such as linearization and independence of random variables. In many algorithms, the loads at each bus are assumed independent and normally distributed [9], which is quite unrealistic for renewable energy and consumer loads. However, dependence of system parameters should be identified by principal component analysis and correlated random variables can be transformed into independent variables. Some authors [10, 11] proposed a linear approach with a rotational transformation to convert variables correlated into uncorrelated.

In Monte Carlo time simulation, a large number of power flows should be run to achieve a good precision. The system optimization (spinning reserve, reactive power, optimal planning,…) usually requires an lengthy iterative process.

VI. Markov Chains in Stochastic Power Flow

One important contribution of this article is the use of cases for describing distributed generators with non discrete operational states. In wind or solar energy, it is not practical to take into account each single wind turbine in the simulation of a big power system. The use of cases along with its frequency of occurrence is a compact way to condense the information of turbines' operational point due to an uncontrollable primary energy, geographically related.

Markov chains are very adequate for handling transitions between states (for example, the change from available to unavailable operational state and vice versa). The main drawback of using only a reduced number of cases is that they must be chosen so that all significant operational states are included in the set. Some cases must be included because they happen very frequently (states with high probability) whereas others can be rare but they can harness system stability. Therefore, no-generation and full generation should be included as states.

Each combination of system states can be solved with a regular power flow and its probability and time variability can be obtained from the DTMC (Discrete Time Markov Chain). If the final grid state is dependent on the previous state, a continuation power flow should be run for each realizable state transition (squaring the number of required power flows).

The joint probability distribution of random variables is implicit in transition matrix **P**, which is estimated from real data or from physical models. This is a desirable feature, since many statistical grid methodologies [12] suppose that random variables are not correlated (independent variables) whereas renewable generation is quite correlated in small geographic areas and loads are also quite correlated.

If each load and generator are discretized, for instance, into four states, the number of combinations are 4^n, where n is the number of stochastic variables considered. If the number of stochastic variables n is big, grouping highly related observations is necessary.

The combined use of discretization and cluster analysis allow to reduce the number of power flow runs compared to standard Monte Carlo Simulation [13]. The discretization and classification errors decrease considering more states.

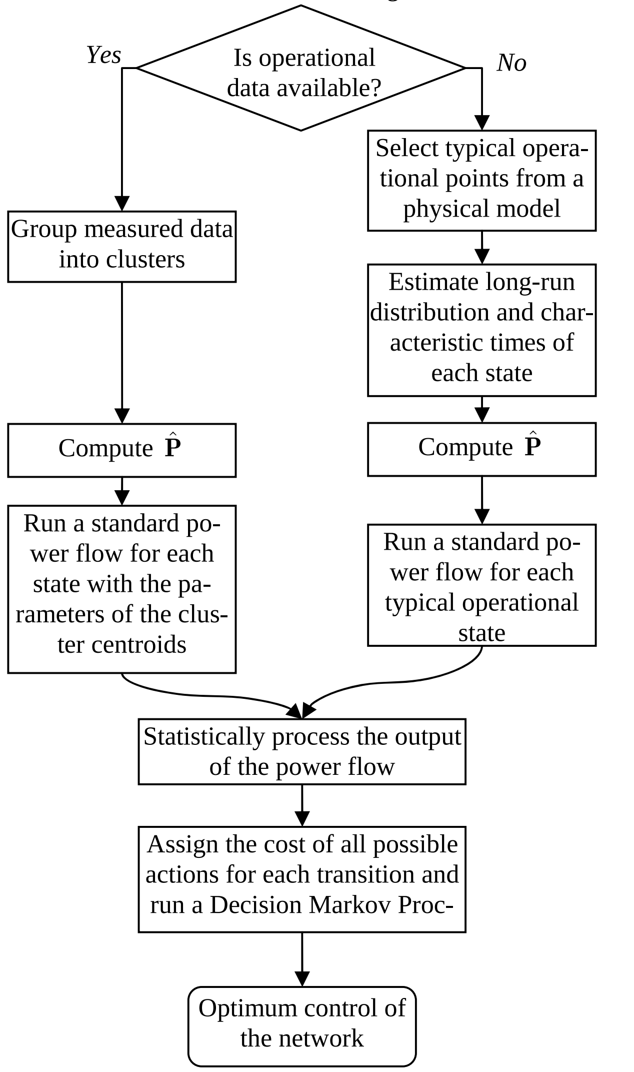

Fig. 2: Work flow for the proposed model .

Some statistical computer packages select the number of cases comparing the decrease ratio of the classification error when the number of states m are increased (i.e., including a new group in the data clustering process) [14]. Therefore, the number of states can be selected depending on the desired classification precision, the non-linear behavior of the electric grid (plausible topological changes in the grid, voltage collapse) and the data available to adjust the model parameters.

If there is enough data, the system state can encompass load and generation. Load is very weakly correlated with wind and wind generation and load can be regarded as independent random variables. The load is dependent of daylight and therefore, solar power is partially correlated to load. In the example of the following part, load and generation would be modeled as non-related Markov Chains and classified independently. The possible combinations of load and wind generation states are $N_{load} \cdot N_{wind}$ and tensor algebra can be used to compute efficiently the properties of the total system (see chapter 9 of [15]). Note that if load and generation are expressed in their respective canonical basis, the combined matrix is a diagonal with the eigenvalues of load and generation. Since $\hat{\mathbf{P}}$ is the matrix $\mathbf{P}$ estimated from data, the probability of $\hat{\mathbf{P}}$ having two or more eigenvalues exactly the same tends to zero for increasing sets of data. Therefore, $\hat{\mathbf{P}}$ is diagonalizable in practice. Thus, the space requirements are proportional to the number of states and the matrix operations are trivial.

Since states of power generation are treated as Markov Chains, the variability of the load will be modeled with Markov States also. Therefore, the generation and consumption patterns are classified in a limited number of states, which are equivalent to transform a multidimensional continuous system into a discrete one. This makes the system tractable and it allows to obtain not only the probability density functions, but also the time variability. Therefore, it is possible to estimate the number of changes in tap changers, in capacitor banks and in the topology of the network.

To sum up, this model requires running just as many power flows as states has the system and it allows to derive easily and rigorously the probability of events. Each case can be solved with a standard power flow, considering non linear elements such as topological changes in the grid that depend on the system loads and generators.

The moments of random variables such as line power flow, generation and voltages can be computed directly from the probabilities of the cases. The continuous distribution of the network can be easily obtained from the cumulative density, adjusting an interpolating function to the case points (the probability density function is the derivative of the interpolating function).

Moreover, the allocation of spinning reserve due to wind power can be done using a Markov Decision Process. These processes can compute the optimum spinning reserve policy from the probability of wind generation variation, the cost of running the reserves and the eventual cost of insufficient reserve.

VII. SIMPLIFIED MODEL OF A WIND FARM TO ACCOUNT ACTIVE AND REACTIVE LOSSES INSIDE THE FARM

The wind farm model employed in this section is based in [16, 17], where a fourth-pole equivalent representation is obtained from the electrical elements, the distributed layout of the turbines, the stochastic nature of power output and small-signal analysis of the grid.

In this paper, an approximate representation with a shunt admittance and series impedance will be used to simplify the analytic expressions .

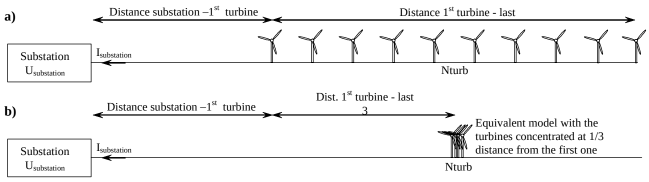

Fig. 1: Original and concentrated model of a MV circuit in a wind farm.

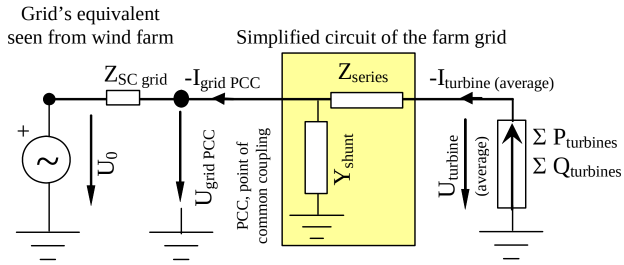

Fig. 2: Model of the farm using a fourth pole realization.

In the analyzed wind farms, the wind turbines are operated at a constant leading angle near $\varphi_0 \sim 0$ at low voltage (LV) side to increase reactive bonus. Therefore, the reactive power Q show a quadratic relationship with active power P due to series inductances and shunt stray capacitance of cables.

$$Q_{WT} \approx P_{WT} Tan(\varphi_0)$$

$$Q_{PCC} \approx -B_{shunt} U_{PCC}{}^2 + \Sigma Q_{WT} + X_{series} \frac{\Sigma P_{WT}{}^2 + \Sigma Q_{WT}{}^2}{U_{PCC}{}^2} \approx \quad (6)$$

$$\approx -B_{shunt} U_{PCC}{}^2 + \Sigma P_{WT} Tan(\varphi_0) + X_{series} \left(1 + Tan^2(\varphi_0)\right) \frac{\Sigma P_{WT}{}^2}{U_{PCC}{}^2}$$

$$P_{PCC} \approx \Sigma P_{WT} - R_{series} \frac{\Sigma P_{WT}{}^2 + \Sigma Q_{WT}{}^2}{U_{PCC}{}^2} - G_{shunt} U_{PCC}{}^2 \approx \quad (7)$$

$$\approx -G_{shunt} U_{PCC}{}^2 + \Sigma P_{WT} - R_{series} \left(1 + Tan^2(\varphi_0)\right) \frac{\Sigma P_{WT}{}^2}{U_{PCC}{}^2}$$

Where

ΣP_{WT} = sum of active power of all turbines (positive when generating)

ΣQ_{WT} = sum of active power of all turbines (positive if generators behave inductively)

P_{PCC} = Active power injected at PCC

P_{PCC} = Reactive power injected at PCC

R_{series} and X_{series} are the real and imaginary part of Z_{series}, i.e. the resistance and reactance of the series equivalent.

G_{shunt} and B_{shunt} are the real and imaginary part of Y_{shunt}, i.e. the shunt conductance and susceptance.

For instantce, Fig. 3 shows the quadratic PQ relationship for a wind farm during one year (15 minutes measures). Its series inductance is about 16 % p.u. due to cable impedances and turbine (U_{cc}=5,8 %) and farm transformers (U_{cc}=7,5 %). The graph is scattered since U_{PCC} was variable and the value of φ_0 was adjusted at the end of each month to obtain the maximum reactive bonus according to Spanish tariff. The other two wind farms show similar relationships.

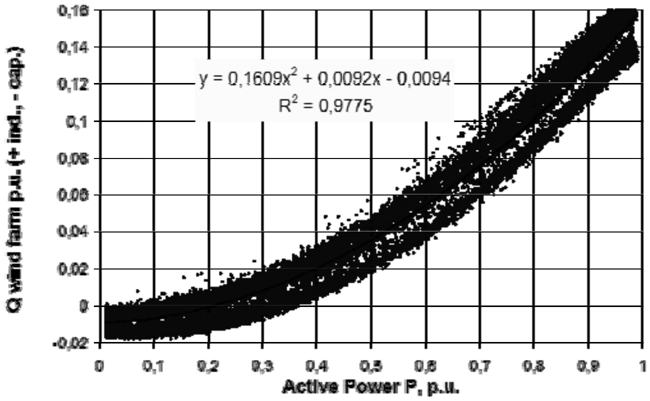

Fig. 3: PQ relationship of a wind farm at 220 kV node during one year

Even though the voltage inside the farm varies, it is expected to be near to assigned value at normal operation ($U_{turbine}$ ~ 1 p.u.). This simplification is only a small source of uncertainty of the model since Z_{series} are expected small p.u. (around 0.12 p.u.) and Y_{shunt} is expected to be big (at least 20 p.u.). Standard UNE 206005 [18] assess the reactive power ability of wind farms at $U_{turbine}$ = 0,95 p.u., 1 p.u. and 1,05 p.u.

The parameters R_{series}, X_{series}, G_{shunt} and B_{shunt} of (6) and (7) can be derived from measures or from simulations at calm (P_{WT} = 0, Q_{WT} = 0) and full power with unity power (P_{WT} = 1 p.u., Q_{WT} = 0) with 1 p.u. voltage at PCC:

$$\begin{aligned} G_{shunt} &= -P_{PCC}\big|_{P_{WT}=0,\, Q_{WT}=0} \\ B_{shunt} &= -Q_{PCC}\big|_{P_{WT}=0,\, Q_{WT}=0} \\ R_{series} &= 1 - P_{PCC}\big|_{P_{WT}=1,\, Q_{WT}=0} - G_{shunt} \\ X_{series} &= Q_{PCC}\big|_{P_{WT}=1,\, Q_{WT}=0} + B_{shunt} \end{aligned} \tag{8}$$

Taking into account the lines that connect the wind farms and the present unity power factor regulation at low voltage generator output, the reactive power of the three wind farms at PCC is:

$$Q_{PCC} \approx -0{,}008\, U_{PCC}^{2} + 0.0057\, \Sigma P_{WT} + 0.1537 \frac{\Sigma P_{WT}^{2}}{U_{PCC}^{2}} \tag{9}$$

where ΣP_{WT} is the average per unit power of the wind turbines of the three farms. Note that (9) is estimated from nominal values of wind farm project whereas Fig. 3 (and similar graphs for the other two wind farms) are measured. The discrepancies are due to differences on real parameters compared to the values assumed, voltage at nodes of the wind farm bellow 1 p.u., operation of wind turbines with power factor below unity and model approximations.

Stand-by losses are smaller than the resolution of a standard power meter, making it difficult to guest from measures. Therefore, active losses are computed from network and transformers parameters.

$$P_{PCC} \approx -P_{\substack{\text{stand-by}\\ \text{losses}}} + 0{,}9819\, \frac{\Sigma P_{WT}^{2}}{U_{PCC}^{2}} \approx 0{,}9819\, \frac{\Sigma P_{WT}^{2}}{U_{PCC}^{2}} \tag{10}$$

In this case, the equivalent parameters of the wind farm interior network are:

$G_{shunt} \approx 0$

$B_{shunt} \approx 0{,}008\ p.u$

$R_{series} \approx 1 - 0{,}9819 - 0 = 0{,}1781\, p.u.$

$X_{series} \approx 0{,}1537 + 0.008 = 0{,}1545\ p.u.$

$\varphi_0 \approx ArcTan(0{,}0057) \approx 0{,}0057\ rad$

VIII. ESSENCE OF THE NEW APPROACH TO PROBABILISTIC POWER FLOW

In essence, stochastic load-flow (SLF) studies assumes that the long-term nodal generation and load vector varies about an expected operating point. The SLF algorithm is easily built from existing state-estimator algorithms [19], but the drawback of the SLF is that it handles only Gaussian nodal probability density function (PDF) data for practical system sizes.

Another approach, commonly referred to as the probabilistic load-flow (PLF) algorithm, uses linear or quadratic approximations of the network behavior. For realistic system sizes, independence of nodal power injections must be assumed in order to be able to apply convolution techniques [20].

The new approach proposed in this paper does not rely on convolution, independence of random variables or linear behavior of the power system. The new method does rely on the fact that power injections are (highly) related and some patterns can be noticed.

The parameters that affect demand curve are well established (week day or bank holyday, season, time of the day, weather temperature, type of consumers, …). Wind generation and other types of dispersed generation show strong links due to geographical and meteorological links. Therefore, the load and the disperse generation can be classified into a (reduced) set of behavior patterns.

Each combination of load and generation patterns represents the typical operation of the power system during some periods. Therefore, a standard (deterministic) power flow can be employed to solve each typical operation. Afterwards, a statistical analysis can be carried out to measure system performance and to optimize the control or the design of the system.

The essence of this new method is a Monte Carlo analysis where the cases are not randomly generated. First, data is classified to select the most representative cases to be simulated, giving further insight in the relationship of the players of the power system (i.e. data mining). Then, a conventional power flow is run with the values of the center of the class (if the previous state of the network is influential, a continuation power flow can be run for each pattern transition). Finally, the results of the simulation are statistically analyzed (usually, to optimize the design or the operation of the network).

Since there are very powerful classification algorithms that can handle efficiently very large amounts of data, the number of time-consuming power flow runs are highly decreased, resulting in an important reduction of computing burden compared to conventional Monte Carlo. Usually, the number of patterns is small enough for all cases to be simulated.

Other advantage of the proposed method is that electrical engineers are used to the simulation of cases (worst scenario, typical seasonal scenario,…). Therefore, this analysis is more familiar to them.

If the generation is not correlated at all, the procedure is still valid but the computing savings decrease. The number of cases to represent the operation of N generators with a given accuracy when they are not related is proportional to N^2 and the method degenerate in conventional Monte Carlo. Recall that if the random variables were independent and the system behavior were sufficiently linear (no parameter violations, bottlenecks or topological changes in the network are expected) techniques such as convolution and two point estimates would be preferable [21].

The modelization of the system variability through Markov chains allows to obtain not only the static system performance but also its slow-dynamic behavior (slow enough for the algebraic power flow equations to remain valid).

Markov chains, in the way they are applied in this paper, can be thought as a system of stochastic differential equations which mimics the measured evolution of loads and generators. The network response to loads and generation evolution is computed based on the power flow equations.

In fact, accounting the previous system state makes possible to include the system operator action, provided it can be specified mathematically (for example, with a set of fuzzy rules based on expert knowledge).

IX. CONCLUSION

The classification of states can be based on power output, "unperturbed wind speed" and wind speed prediction, depending on available data and aim of the wind farm model. The performance matrix in Standard IEC 61400-12-3 can be used as emission matrix to relate wind and power in a wind farm using a Hidden Markov Model. The wind farm model can be used also as time interpolation or to guess if there is an outlier in the state (a switching event).

The basic workflow to compute a stochastic power flow based in Markov Model using a regular program is presented. A simplified, steady state, quadratic model of the wind farm is shown for justifying the approximation of networks to PQ nodes and the interpolation between states. This quadratic model can be used also to estimate the reactive power for steady state.

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