Reactive Power Injection Strategies For Wind Energy Regarding Its Statistical Nature

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Abstract— This work studies reactive power strategies for wind turbines and wind farms. Previous Spanish regulation stated unity power factor (P.F.) as target. New regulation introduced in RD 436/2004 rewarded up to 8% with P.F. < 0,95 inductive (on low-load hours) and P.F. < 0,95 capacitive (on peak hours).

A better utilization of the infrastructure can be attained considering the reactive power generation availability even in calm periods –depending on the reactive power compensation scheme implemented in the wind farm-. This can lead to a voltage support strategy in a nearby node, especially if the voltage at that node can be estimated despite tap changing transformers.

The effect of some control strategies in nearby nodes are quantified statistically regarding the stochastic nature of wind power.

Index Terms— wind energy, power flow, reactive power, stochastic model.

I. INTRODUCTION

According to the Spanish regulation in RD 436/2004 [1], current regulation rewards the control of power factor to generators in the especial regulation (most renewable energy and cogeneration). Remunerated reactive power compensation is based on a power factor band depending on the low-load, peak or medium-load classification of the interval.

Distribution networks are usually limited, not for the maximum thermal current, but for the limits in voltage variation (especially at the end of the line) [2]. Because of that, distribution companies prefer distributed generation to be connected to higher voltage levels, where its impact in voltage distribution is very small. However, connecting to a higher voltage level increases the cost of the plant. In case the wind farm is able to control reactive power absorption or generation, voltage can be fixed, avoiding higher voltage connection [3, 4].

The new Spanish regulation is an improvement from the previous one that stated unity power factor at any time (at peak hours the farm is rewarded to be capacitive and at low-loads the farm is rewarded to be inductive). However, the reactive bonus is obtained if a power factor is accomplished. Therefore, reactive power injection depends on wind and grid support is low at low active power (i.e. low winds).

A better utilization of the infrastructure can be attained considering the availability of reactive power at low active generation or even with no active power generation (many wind turbine technologies can generate more reactive power at low active power or even, without generating active power). As the capacity factor of wind farms is usually low, this would increase the exploitation of the infrastructure.

The suitable system operation may require a set point from the control centre or measures at other points in the grid. Indeed, the power factor band regulation of RD 436/2004 may be regarded as a rough estimation of the load depending on the low-load, peak or medium-load time classification.

If online communication from control centre is not available, the set point of reactive power could be scheduled from the estimation of grid state and the forecast of near loads (based on in time of the day, day of the week, working day, weather, etc.). Moreover, the voltage at the supported node can be estimated at each turbine without the requirement of a centralised control. The wind farm model allows estimating power injection from other turbines based on its power output and its wind direction. With the estimation of power injection of the wind farm and, eventually, nearby wind farms it is possible to compute a rough estimation of voltage at the supported node. The change of taps in transformers that connects the wind farm to the grid can be detected and included in the estimation of the supported voltage node.

II. WIND SITE RESOURCE

Wind availability can be characterized through its measured cumulative distribution function. For estimated calculations, a Weibull distribution can be adjusted to measured data through the *shape* and *scale* factors. Its cumulative distribution function CDF is

Probability (Wind at site $\leq w_{\rm S}$) = CDF_{wind}($w_{\rm S}$) =

= 1 -
$$Exp$$
 -(w_{WT} / scale)^{shape}

median = scale
$$[Ln(2)]^{1/\text{shape}}$$

m

ean wind =
$$\mu_{wind}$$
 = scale $\Gamma\left(1 + \frac{1}{\text{shape}}\right)$ (1)

where Γ is the gamma function.

This model is precise enough for an approximate study of the grid in steady state.

However, the above model doesn't give information about the time variability of wind. The interaction between the wind fluctuations and the turbine is very complex and a thorough model of the turbine, generator and control system is needed for assess their influence in power output [5, 6, 7].

However, fast wind gusts and turbulence are smoothed in the output of the wind farm due to the fact that faster fluctuations have lower spatial correlation [8]. Cyclic time variations are usually characterized from power spectral density of power [9]. Fast fluctuations affect very weak networks with flicker concerns [10] (some authors also studied the synchronisation of blade passing the tower of the turbine [11]).

Non-cyclic variations are usually regarded as random component of the wind. They concern the control (short horizon prediction) and the forecast (long horizon prediction). Artificial Intelligence techniques as neural networks, fuzzy logic and advanced filtering have been used. Time series are quite popular for modelling since its parameters and its properties can be easily estimated [12, 13].

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III. TURBINE POWER CURVE

The power curve relates wind at hub height and power output [14]. Turbine manufacturers provide the curve along with its uncertainty. Fig. 1 shows a power curve from a pitch controlled turbine. The uncertainty is small, except at cut-off wind speeds.



Fig. 1: Example of measured power curve (from [14]).

Paper [15] normalize power curves to ease the optimization of energy production. In this study, a simpler power curve will be employed to be able to derive analytic expressions. The power curve $P_{WT}(w_{WT})$ of (2) is characterized by cut-in, cut-off wind speeds and the wind speeds where power speed is 25% and 75% of turbine rated power. It fits well to pitch and active stall wind turbines, but it is less accurate for passive stall turbines. Those turbines can be modelled more precisely following an analogue procedure to the one shown in this work.

 $P_w(w_{\rm WT}) =$ (2) $1 - Tanh\left(\frac{w_{\rm WT} - \frac{w_{25\%} + w_{75\%}}{2}}{\frac{w_{25\%} - w_{75\%}}{Ln(3)}}\right)$ $W_{cut} < W_{WT} < W_{cut}$ 0 elsewhere Power curve 1 Power Output (p.u.) 0.8 0.6 0.4 0.2 0 5 10 15 20 Wind Speed (m/s) 0 25 30 Fig. 2: Simplified power curve from (2) for example of Annex I.

The cumulative density function is derived in (3) and it is plotted in Fig. 3 for the data shown in Annex I (a Weibull wind distribution with parameter shape = 2, known as Raleigh Distribution).

$$\begin{aligned} &\Pr(Power < p_{WT}) = \Pr(Wind < P_w^{-1}(p_{WT})) + \Pr(Wind > w_{cut-off}) = \\ &= CDF_{Wind}(P_w^{-1}(p_{WT})) + 1 - CDF_{Wind}(w_{cut-off}) = \\ &= 1 + Exp\left[-(w_{cut-off} / scale)^{shape}\right] - \\ &- Exp\left\{-\left[\frac{w_{25\%} + w_{75\%}}{2 scale} + \frac{w_{25\%} - w_{75\%}}{Ln(3) scale} ArcTanh\left(1 - \frac{2p_{WT}}{P_{noninal}}\right)\right]^{shape}\right\} \end{aligned}$$
(3)



Fig. 3: Plot of CDF_{Power Output} for the wind turbine of the example of Annex I.

The statistical parameters of this distribution have been derived analytically. The errors introduced in the approximations needed to achieve expressions 4 to 6 (typically 1% to 4%) are far lower than the errors due to the simplistic models of the wind and the turbine.

The median of power output can be computed using (4). The median is approximately proportional to wind scale parameter.

$$median \approx \frac{P_{nominal}}{w_{75\%} - w_{25\%}}.$$
(4)

$$\left[0.28015 \text{ scale} \left(0.69315^{1/\text{shape}} \right) - 0.51124 w_{25\%} + 0.23109 w_{75\%} \right]$$

Mean wind speed and power output of wind farms with similar wind distributions and power curves- are linearly related to a high degree, as can be seen in Fig. 4. However, the adjusted straight line does not cross the origin. $(\overline{P}_{WT} \sim a + b \overline{w}_{WT}; a \neq 0).$



Fig. 4: Plot of average Power Output versus average wind speed at hub height of the wind farm for the example. P_{-}

$$mean \approx \frac{-nomman}{360} \cdot \left[178 \ Exp\left(-\left(\frac{w_{25\%}}{\text{scale}}\right)^{\text{shape}} \right) + 238 \ Exp\left(-\left(\frac{w_{75\%}}{\text{scale}}\right)^{\text{shape}} \right) + -72 \ Exp\left(-\left(\frac{w_{25\%} + w_{75\%}}{2 \ \text{scale}}\right)^{\text{shape}} \right) + 23 \ Exp\left(-\left(\frac{w_{cut-in}}{\text{scale}}\right)^{\text{shape}} \right) - -367 \ Exp\left(-\left(\frac{w_{cut-off}}{\text{scale}}\right)^{\text{shape}} \right) \right]$$
(5)

The standard deviation of power output (6) reaches the maximum at mean hub wind speeds that correspond to half the power, $w_{50\%}$, as can be seen in Fig. 5. In those cases, the probability density function (pdf) of power output is Ushaped, with two modes at the extremes, as can be seen in Fig. 6.



Fig. 5: Plot of σ $_{Power \ Output}$ versus average wind speed at hub height of the wind farm for the example.

The probability density function can be obtained deriving the CDF _{Power Output} respect to power output.



Fig. 6: Plot of PDF Power Output for example of Annex I.

The uncertainty of the power curve is bigger near cut-off and between cut-in and rated power, as can be seen in Fig. 1. The uncertainty of power curve would be considered in the wind farm power curve in the next section.

IV. WIND FARM POWER CURVE

The objective of this point is to get a simplified representation of the wind farm enough accurate to use it in the statistical model [16, 17].

Standard IEC 61400-123 [18] shows a detailed method to compute the wind farm power output from data of the meteorological mast. The description of the wind farm operation consists of performance matrix **M** indicating the declared power output expected of the wind farm for pairs of wind speed and wind direction values.

In a farm, the distribution of the speed among the turbines is quite dependent of wind direction and that is why the prospective standard IEC 61400-123 utilizes a matrix dependent on wind direction.

This matrix can be expected to be produced as an integral part of the wind farm design process. The information needed to compute this matrix can consist of:

- Long-term climate information at a reference position.
- Turbine performance characteristics.
- Wind flow models for assessing topographic, land cover and turbine wake characteristics.
- Electrical data to compute losses inside the grid of the farm.

Each matrix element $m_{i,j}$ is the expected power output for the wind speed w_i and direction θ_j corresponding to the bin i, j. Therefore, the estimated power \hat{P}_{output} can be computed from the wind speed and direction bin probability, $\Pr(w_i, \theta_j)$.

$$\widehat{P}_{output} = \sum_{j}^{M} \sum_{i}^{N} m_{i,j} \cdot \Pr\left(w_{i}, \theta_{j}\right)$$
(8)

The distribution of power output can be found, for example, using the relationship

$$Pr(p_{k} \leq P_{output} < p_{k+1}) = \sum_{i}^{M} \sum_{j=1}^{N} if(p_{k} \leq m_{i,j} < p_{k+1}, \quad \Pr(w_{i}, \theta_{j}), \quad 0)$$
(9)

To be able to obtain an analytic expression, a simpler model will be employed based on the standard deviation σ_w of wind speed inside the wind farm. Therefore, the considered characteristics of the wind would be its spatial average w_{wf} and its standard deviation σ_{wf} inside the wind farm in ten minute intervals. If there is no data about the wind distribution inside the farm, σ_{wf} can be estimated from the standard deviation of turbine efficiency (from microsite optimization) or from the variance of energy output that is usually available at SCADA (average power and average wind speed show a behaviour similar).

The main effect of the wind distribution inside the farm would be:

• Decrease of wind farm w_{wf} average speed from undisturbed wind speed value w_S of the site model (section III of this paper). If performance matrix is not available and there is no more information, it is reasonable to multiply the undisturbed value by an efficiency factor η_{wf} that would be around 90-95%, depending on the wake effects [19].

$$w_{wf} = \eta_{wf} w_S \tag{10}$$

 Smooth the power curve since each turbine experience a slightly different speed. This smoothing is evident at cut-off speed.

The power output of the wind farm would be the sum of the output of the turbines (less the grid losses that would be computed in next section).

If wind speed distribution inside the farm is known, the power output distribution of the turbines would be:

$$pdf_{Power}(p_{\rm WF}) = \frac{1}{\left| dP_{\rm w} / dw \right|} pdf_{wind \ speed}(w_{\rm WF}) \bigg|_{w=P_{\rm w}^{-1}(p)}$$
(11)

The average power output for a given mean wind speed is the cross correlation of the power curve and the wind distribution:

$$\overline{P}_{Power}_{output}(w) = \int_{-3\sigma_{wf}}^{3\sigma_{wf}} p df_{wind}_{speed}(\tau) \cdot P_{w}(w+\tau) d\tau$$
(12)

The average power for a normal distribution cannot be computed analytically unless some approximations are made. However, if slope of power curve is quite steady in the interval of $w_{wf} \pm \sigma_{wf}$, the average power output can be approximated as $\hat{P}_{Power output}(w) \approx P(w)$. This is a good approximation except for the cut-off speeds, when the disconnection of turbines causes an abrupt change of the power curve. For such cases, the wind farm power curve can be characterized with two extra parameters that reflect how abrupt the disconnection of the turbines is:

$$P_{wf}(w_{S}) = \frac{P_{\text{nominal}}}{2} \left[Tanh \left(Ln(3) \frac{w_{S} - \frac{w_{25\%} + w_{75\%}}{2}}{w_{25\%} - w_{75\%}} \right) - Tanh \left(Ln(3) \frac{\eta_{wf} w_{S} - w_{cut off}}{\Delta w_{off}} \right) \right]$$
(13)

 Δw_{off} is the difference between the wind speed when there is a 25 % and 75 % of disconnection of the turbine due to high wind.

 $w_{\text{cut-off}}$ is the wind speed when there is a 50% probability of the turbine to shut down.

 η_{wf} is the farm mean efficiency factor.





Fig. 7: Power curve of the wind farm (solid) and the turbine (dashed) for the example of annex I.

If all turbines of the wind farm are the same model, the power curve normalised "per unit" is analogous for the grid and for the turbine.

Near cut-off wind speed, the power curve is highly non linear. Also, the uncertainty of power curve is greater there. In such cases, the power show greater variability. In such wind regime, the turbines would gradually shut down starting from those more exposed.

$$\begin{aligned} & \Pr(Power_{WF} < p_{WF}) = & (14) \\ &= \Pr(Wind_{S} < P_{WF}^{-1}(p_{WF})) + \Pr(Wind_{S} > P_{WF}^{-1}(p_{WF})) = \\ &= \operatorname{CDF}_{wind}(P_{WF}^{-1}(p_{WF})) + 1 - \operatorname{CDF}_{wind}(P_{WF}^{-1}(p_{WF})) = \\ &= 1 + \operatorname{Exp}\left\{ -\left[\frac{w_{cut \ off}}{\frac{w_{cut \ off}}{Ln(3)}} \frac{ArcTanh\left(1 - \frac{2p_{WF}}{P_{nominal}}\right)}{scale} \right]^{shape} \right\} - \\ &- \operatorname{Exp}\left\{ -\left[\frac{\frac{w_{25\%}}{2} + \frac{w_{75\%}}{Ln(3)}}{\eta_{wf} \ scale} \right]^{shape} \right\} \end{aligned}$$



Fig. 8: Plot of PDF Power Output for the wind farm power output of Annex I.

$$\begin{aligned} mean &\approx \frac{P_{\text{nominal}}}{360} \left[-178 \ Exp \left(-\left(\frac{2 \ w_{cut-off} + \Delta w_{off}}{2 \ \eta_{wf} \ \text{scale}}\right)^{\text{shape}} \right) + \\ +23 \ Exp \left(-\left(\frac{w_{cut-in} / \eta_{wf}}{\text{scale}}\right)^{\text{shape}} \right) - 23 \ Exp \left(-\left(\frac{w_{cut-off} / \eta_{wf}}{\text{scale}}\right)^{\text{shape}} \right) + \\ +178 \ Exp \left(-\left(\frac{w_{25\%} / \eta_{wf}}{\text{scale}}\right)^{\text{shape}} \right) + 238 \ Exp \left(-\left(\frac{w_{75\%} / \eta_{wf}}{\text{scale}}\right)^{\text{shape}} \right) \\ -72 \ Exp \left(-\left(\frac{w_{25\%} + w_{75\%}}{2 \ \eta_{wf} \ \text{scale}}\right)^{\text{shape}} \right) + 72 \ Exp \left(-\left(\frac{w_{cut-off} / \eta_{wf}}{\text{scale}}\right)^{\text{shape}} \right) - \\ -238 \ Exp \left(-\left(\frac{2 \ w_{cut-off} - \Delta w_{off}}{2 \ \eta_{wf} \ \text{scale}}\right)^{\text{shape}} \right) \right] \end{aligned}$$

$$(15)$$

The uncertainty of wind at the site is σ_s (the uncertainty is the deviation of the parameter measured or estimated). The value of the combined standard uncertainty is equal to the standard deviation of the measure, that is a stochastic variable [20].

The relationship between the standard deviation of wind farm speed σ_{wf} and power output of a single turbine σ_{pof} due to wind deviation can be approximated by:

$$\sigma_{\substack{\text{power}\\\text{power}\\\text{output}}}(p_{\text{WF}}) \approx \left| \frac{dp_{\text{WF}}}{dw_{\text{WF}}} \right| \cdot \sigma_{\text{WF}} \right|_{w = P_{w}^{-1}(p)} = \frac{P_{\text{nominal}}(w_{75\%} - w_{25\%})\sigma_{\text{WF}}}{p_{\text{WF}}\left(P_{\text{nominal}} - p_{\text{WF}}\right)Ln(9)}$$
(16)

The deviation of the whole farm power output is not the sum of individual turbine deviations since some degree of cancellation is achieved. Unless more detailed data are available, it is reasonable to expect a normal distribution of wind speeds at the farm. If wind is distributed normally independent among n turbines of a farm, the deviation is only \sqrt{n} times the deviation of a single turbine, instead of n.

$$\sigma_{farm}_{power}(p_{\rm WF}) \approx \sqrt{n} \, \sigma_{turbine}_{power}(p_{\rm WF}) = \frac{P_{nominal}(w_{75\%} - w_{25\%}) \, \sigma_{_{\rm WF}}}{\sqrt{n} \, p_{_{\rm WF}}\left(P_{nominal} - p\right) Ln(9)}$$
(17)

The standard deviation of power output of the farm σ_{po} due to wind deviation at cut-off speeds can be approximated by:

$$\sigma_{farm \ power}\left(p_{\rm WF}\right) \approx \frac{P_{\rm nominal \ farm} \ \Delta w_{off}}{\sqrt{n} \ p\left(P_{\rm nominal \ farm} - p_{\rm WF}\right) Ln(9)} \sigma_{wf} \tag{18}$$

The standard deviation of power output of the farm in function of the undisturbed wind speed of the site w_s is:



Fig. 9: Power curve of the wind farm (dots) and the uncertainty of the power for the example of annex I.

The uncertainty of power curve near cut-off usually partially accounts for this deviation since the wind varies during the 10 minute interval used for the manufacturer to compute the power curve.

V. MODEL OF THE ELECTRICAL GRID OF THE FARM

The approach followed in this section is based in [21], where a simplified model of the wind farm is derived based on the fourth-pole equivalent representation of the electrical elements, the distributed layout of the turbines, the stochastic nature of power output and small-signal analysis of the grid. The uncertainties of the approximations made in the model are also assessed there. The overall system uncertainty is barely affected by this representation since it is precise enough and the grid behaviour is much more deterministic than the wind and power curves.

The approximate method of the moments is widely accepted in electrical engineering. Using that approach, the turbines can be concentrated in some points, as it is pictured in Fig. 10. Due to the fact that the turbines inside a wind farm have similar power output and voltage, a "concentrated model" can be used for accounting power losses in shunt admittances and series inductances.

The behaviour of turbines resemble PQ nodes because its efficiency is high, voltage is near nominal value for usual operation and because reactive control tries to reach control target (usually certain power factor).



Fig. 10: Original and concentrated model of a MV circuit in a park.

Paper [21] computes the model parameters form resistance, capacity and reactance of each element, obtaining the farm equivalent transmission matrix shown in

the Fig. 9. However, it is more convenient to estimate the parameters from power flow solutions.



Fig. 11: Model of the farm using its transmission matrix.

The farm equivalent can be regarded as Π or T equivalent. However, an approximate representation with a shunt admittance and series impedance will be used to simplify the analytic expressions. If a more precise model of the farm is needed, a full Π or T fourth pole equivalent can be used (another simulation case would be needed to estimate the extra parameter).



Fig. 12: Model of the farm using a fourth pole realization.

In this article, the electrical parameters of the farm will be expressed in per unit using the nominal power of the wind farm as base.

The relation of power flow at turbines and at point of common coupling PCC can be easily derived.

$$P_{PCC} = P_{WT} - R_{series} \frac{P_{WT}^{2} + Q_{WT}^{2}}{U_{PCC}^{2}} - G_{shunt} U_{PCC}^{2}$$
(20)

$$Q_{PCC} = Q_{WT} - X_{series} \frac{P_{WT}^{2} + Q_{WT}^{2}}{U_{PCC}^{2}} + B_{shunt} U_{PCC}^{2}$$
(21)

Where

 $P_{WT} = \Sigma P_{turbines} =$ sum of active power of all turbines $Q_{WT} = \Sigma Q_{turbines} =$ sum of active power of all turbines $P_{PCC} =$ Active power injected at PCC $P_{PCC} =$ Reactive power injected at PCC

 R_{series} and X_{series} are the real and imaginary part of Z_{series} , i.e. the resistance and reactance of the series equivalent. G_{shunt} and B_{shunt} are the real and imaginary part of Y_{shunt} , i.e. the shunt conductance and susceptance.

In case of fixed capacitors that are always connected, it is more precise to compute them in B_{shunt} . Also, if the maximum supply or drain of reactive power is being studied, all the capacitors and inductances shunt connected should be included in B_{shunt} . In other cases, the automatic reactive compensation must be included in Q_{WT} .

Even though the voltage inside the farm varies, it is expected to be near to assigned value on normal operation $(U_{turbine} \sim 1 \text{ p.u.})$. This simplification is only a small source of uncertainty of the model since Z_{series} are expected small p.u. (around 0.12 p.u.) and Y_{shunt} is expected to be big (at least 20 p.u.). Standard UNE 206005 [22] assess the reactive power ability of wind farms at $U_{turbine} = 0,95$ p.u., 1 p.u. and 1,05 p.u. This standard states a method to compute farm power losses that is equivalent to the one presented in this section.

The new method to obtain the farm equivalent consists on simulate the wind farm with a power flow program at two power levels and to solve the parameters R_{series} , X_{series} , G_{shunt} and B_{shunt} from equations (20) and (21). For 1 p.u. voltage at PCC and power simulations at calm ($P_{WT} = 0$, $Q_{WT} = 0$) and full power with unity power ($P_{WT} = 1$ p.u., $Q_{WT} = 0$), the parameters are:

 $G_{shunt} = -P_{PCC}|_{P_{WT}=0, Q_{WT}=0}$ $B_{shunt} = Q_{PCC}|_{P_{WT}=0, Q_{WT}=0}$ $R_{series} = 1 - P_{PCC}|_{P_{WT}=1, Q_{WT}=0} - G_{shunt}$ $X_{series} = -Q_{PCC}|_{P_{WT}=1, Q_{WT}=0} + Y_{series}$ (22)

VI. MODEL OF NEARBY WIND FARMS

The influence of near wind farms should be taken into account because their active power output are quite correlated (they show a similar behaviour).

A simple linear correlation can be enough precise for grid studies of near wind farms. Far away wind farms show generally complex relationships and have low correlation coefficients. But those farms are expected to interact less with the studied wind farm. Therefore, a linear correlation of the farms is enough in most cases (the more influencing wind farms are better modelled).

If grid parameters vary linearly with the power output of the analyzed wind farm, the average effect of other wind farms would be approximately computed using average power for the given selected power level of the studied farm (a linear function, applied to a stochastic variable, also transform linearly the expected value and standard deviation of that variable). For example, voltage deviations and power flows are related mainly linearly with power (except near a voltage collapse or very high wind share).

The linear regression of the power predicted in wind farm "j" based on the power of the reference farm "i" is given by:

$$\mathbf{P}_{j} = \mathbf{P}_{j} + b_{j} \left(\mathbf{P}_{i} - \mathbf{P}_{i} \right)$$
(23)

$$b_j = r_{ij} \frac{s_j}{s_i} \tag{24}$$

Where P_j and P_i are the average power output in park "j" (estimated farm) and "i" (reference farm);

 r_{ii} is the experimental correlation coefficient;

 s_i and s_j are the standard deviation of power in farms i and j.

The interaction between reactive power of wind farms must be also taken into account. If the control is a fixed power factor, Q = P tan φ , a linear correlation is also advisable for compute reactive power injection of other wind farms (note that unity power factor is a special case where $\varphi = 0$).

In case of Automatic Voltage Support or other control strategies, the reactive power of the wind farms must be estimated accordingly.

A precise model of interaction can be needed in some studies (topology changes and congestion typically show a non-linear behaviour). In such cases, a Monte Carlo simulation is advisable, where wind power and load are stochastically modelled. This type of study is beyond the scope of this article.

VII. LIMITS ON REACTIVE POWER

The maximum amount of reactive power that can be injected or absorbed are given by:

- Limits provided by the turbine manufacturer. Second edition of IEC 61400-21 will include a section devoted to the reactive power capability and the ability to participate in an automatic voltage control scheme.
- Allowable voltage limits at the turbines. The wind turbine that is electrically farer from PCC will suffer the greatest voltage deviations of the wind farm.
- Allowable current in series elements (electronic converters, lines, transformers, etc).

Turbines inside a wind farm operate at similar power levels and voltage drops are small enough to use the linear relationship:

$$U_{WT} \approx U_{0} + \frac{\left(R_{SC \text{ at } PCC} + R_{series}\right)P_{WT} + \left(X_{SC \text{ at } PCC} + X_{series}\right)Q_{WT}}{U_{0}}$$

$$\Delta U_{worse} = R_{eff}P_{WT} + X_{eff}Q_{WT}$$
(25)

where the parameters R_{eff} y Q_{eff} can be adjusted from a simulated power flow with $P_{WT} = 1$ p.u., $Q_{WT} = 0$ and with $P_{WT} = 0$, $Q_{WT} = 1/3$ p.u. (a simulation with $Q_{WT} = 1$ p.u. can lead to voltage out of range in many cases and the linear aproximation is not valid near voltage collapse).

$$R_{eff} = \frac{R_{sc} + R_{series}}{U_0} = U_{worse} \bigg|_{P_{WT} = 1 \text{ p.u., } Q_{WT} = 0} - U_0$$

$$X_{eff} = \frac{X_{sc} + X_{series}}{U_0} = \frac{1}{3} U_{worse} \bigg|_{P_{WT} = 0, \ Q_{WT} = 1/3 \text{ p.u.}} - U_0$$
(26)

The limit of voltage rise or drop leads to a band of allowed power in the P,Q plane (p.u.). $U = \langle U_{a} + \Delta U \rangle = \langle U \rangle$

$$\Delta U_{\min} < C_0 + \Delta C_{worse} < C_{max}$$

$$\Delta U_{\min} = U_{\min} - U_0 < \Delta U_{worse} < U_{max} - U_0 = \Delta U_{max}$$

$$\Delta U_{\min} < R_{eff} P_{WT} + Q_{eff} P_{WT} < \Delta U_{max}$$

$$Upper voltage limit: \qquad (27)$$

 $R_{eff}P_{WT} + Q_{eff}P_{WT} = \Delta U_{max}$

Lower voltage limit :

$$R_{eff}P_{WT} + Q_{eff}P_{WT} = \Delta U_{min}$$

The locus of excessive current is determined by a circumference of radius S_{max} = $U_{turbine}\,I_{max}\,(p.u.)$



Fig. 13: Operational limits of turbine reactive power Q_{WT} due to excessive voltage deviations and over current at the wind turbine.

The quadratic equations (20) and (21) transforms the P, Q limits of Fig. 11 (at turbine) to the P, Q values that are achievable at PCC. Therefore the straight lines bend a little and the circle is slightly smashed when the power limits are calculed at PCC

VIII. REACTIVE POWER POLICY

Nowadays, most turbines use unity power factor regulation [23]. However, the reactive power injection can achieve some goals:

- Minimize voltage variations at a point in the grid due to the wind farm. This control would lead to a power factor near unity at wind turbines (slightly inductive).
- Stabilize voltage at a nearby point of the network. This control would need to measure the actual voltage at the reference node or, alternatively, an algorithm to estimate voltage there from voltage measured at wind farm.
- Try to compensate reactive needs in the surrounding grid. In fact, this would also minimize power losses in the grid. This strategy would be typically managed by a control centre that measures nearby load consumption, line flows and grid constrains.

IX. CONSTANT POWER FACTOR REGULATION

The voltage variations due to a wind farm with constant power factor would be proportional to active power.

$$\Delta U_{WT} = \frac{R_{eff} + X_{eff} \tan \varphi_{WT}}{U_0} P_{WT} = K_{\varphi} P_{WT}$$

$$R_{eff} + jX_{eff} = \frac{Z_{SC}}{1 + Y_{shunt} Z_{SC}} + Z_{series}$$

$$U_0' = U_0 \frac{1 + Y_{shunt} Z_{series}}{1 + Y_{shunt} Z_{SC}}$$
(29)

The voltage influence of the farm can be cancelled at the wind turbines ($\Delta U_{WT} \approx 0$) or at the PCC ($\Delta U_{PCC} \approx 0$).

If the target is not to influence voltage at a point, the wind farm will behave inductively, that in many scenarios is not a desired scenario.

$$\Delta U_{WT} \approx 0 \implies K_{\varphi} = 0 \implies \tan \varphi = -R_{eff} / X_{eff}$$
(30)

If the target is $\Delta U_{PCC} \approx 0$ then the power factor is determined by the short circuit impedance at the PCC, $\tan \varphi_{PCC} = -R_{SC \text{ at PCC}}/X_{SC \text{ at PCC}}$

If the target is a fixed power factor at PCC, then the value of turbine reactive power Q_{WT} can be determined solving the quadratic equations (20) and (21). For unity power factor at PCC, there must be injection of reactive power at the wind farm that is not proportional to P_{WT} .

$$Q_{PCC} = \frac{U_{PCC} - \sqrt{U_{PCC}^{2} + 4 B_{shunt} X_{series} U_{PCC}^{3} - 4 P_{WT}^{2} X_{series}^{2}}{2 X_{series}}$$
(31)

The effect of a power factor in the voltage profile can be computed taking into account that the voltage deviation due to the farm is proportional to the active power output, $\Delta U_{WT} = K_{\varphi} P_{WT}$. Therefore, the voltage distribution and the power have the same shape as (see Fig. 8)and the scale factor is K_{φ} .

X. AUTOMATIC VOLTAGE CONTROL

The voltage control is difficult to achieve without communication with a control centre. If neither the detection of voltage regulators is made nor there is connection to a monitoring centre, the estimation of voltage at PCC from farm voltage can be fooled. Think in an under voltage scenario, where tap or a topological change in the grid push up the voltage at the wind farm. Then, the automatic voltage regulator (AVR) can make the wind farm to restrain the reactive power injection (or even, to start behaving inductively), increasing the deficit of reactive power in the grid.

Even if there are other loads connected between the wind farm and the reference node (usually, PCC), the state of tap changing transformers or voltage boosters in the line can be accounted. If tap changing transformers are close to the wind farm, the sudden voltage or angle jump can be detected and identified. However, the voltage or angle jump must be significantly bigger than voltage variations due to nearby sudden load variation or connection of nearby farms.

A Bayesian decision tree can be used to detect tap changes. Voltage deviations due to nearby loads can be estimated from statistical data of consumers, even though these data can be difficult to obtain in a de-regulated market (for example, the loads can be estimated from working day classification, month and hour)

One way to compute Q_{WT} to support voltage a net node is to compute $\partial U_{node} / \partial Q_{WT}$ through network simulation at two power levels.

$$\frac{\partial U_{node}}{\partial Q_{wT}} \approx \frac{\Delta U_{node}}{\Delta Q_{wT}} = \frac{U_{node}|_{Q_{WT} = Q_{MAX}} - U_{node}|_{Q_{WT} = 0}}{Q_{MAX} - 0}$$
(32)

$$Q_{WT} = K_{Q_{WT}} \frac{\Delta U_{\text{measured or}}_{\text{estimated at node}}}{\frac{\partial U_{node}}{222}}$$
(33)

 ∂Q_{WT} The weighting factor $0 < K_{Q_{WT}} < 1$ accounts for the

fact that more generators and devices are performing voltage support. This factor must be small if the supported node is electrically far from the wind farm. Otherwise, the turbines would operate very often at maximum reactive power absorption or injection.

XI. SCHEDULED REACTIVE CONTROL

If communication with the system operator (S.O.) is not possible, a schedule of reactive power at PCC based on load is possible. In fact, Spanish regulation RD 436/2004 [1] rewards certain power factor depending on the time of the day and the Spanish region. This is an improvement from past regulation (unity power factor) since there is more correlation between system reactive needs and reactive generation.

However, actual Spanish regulation is based in type 3 classification of the tariff established in OM 12/1/1995 [24]. A clear improvement would be to establish the bonus based on type 5 schedule, where the type of the day (labour, weekend, bank holiday) and the season would be also considered. The improvement would be due to higher correlation between system reactive needs and reactive generation. The increase of control complexity with type 5 schedule is very small since all SCADA have a built in calendar.



Fig. 14: Distribution of voltage deviations at PCC due to the wind farm of Annex I (data corresponding to P.F. at wind turbine 0,95 inductive in blue; 0,95 capacitive in yellow and unity in magenta).

One drawback of power factor discount is that, as active power is random, voltage and reactive power support would be also. Calm and low wind are the more likely states at wind farms, as can be seen from Fig. 6. In such states, the grid support and the use of available infrastructure is low.

Moreover, the reactive power capability of most wind farms is bigger at low active power: many technologies and compensating devices can inject or absorb reactive power when the generator is not connected.

Therefore, other clear *improvement is to compute the reactive bonus on reactive power*, not on power factor. Voltage at PCC for several power factors



XII. REACTIVE POWER UNDER CENTRALIZED CONTROL

The optimum policy for reactive power control must support voltage and try to maintain power losses at low level, avoiding network congestion.

The cost of voltage support, power losses and net congestion can be derived from voltage deviation penalty at border nodes, mean power tariff and costs derived from congestions. An optimum power flow could attain a global optimum considering these factors [25].

Reactive power pricing must be adjusted carefully for the optimum control to be performed.

The availability of reactive power injection is a random variable because it depends on wind. It must be assessed depending on the technology of wind turbines, ancillary reactive devices and wind potential at the site. Even though active and reactive power are related, existing technology allow some level of control freedom. For example, Fig. 15 show the realizable power at a turbine equipped with a full rated converter. The limits on the converter displayed are due to maximum current and maximum voltage at turbine. Other constrains can appear due to internal features of the converter, but they are not considered here (for example, the choke coils can decrease the capacitive capability of the converter, but here is not considered).



Fig. 16: Realizable reactive power at the wind turbine for the example of Annex I.

The probability of being able to inject more than Q reactive power at the PCC can be computed trough the cumulative distribution of power.

$$\Pr(q_{WT} < Maximum Q) = \Pr(Power < Q_{WT MAX}^{-1}(q_{WT}))$$
(34)



Fig. 17: Availability of reactive power injection (capacitive behaviour of the WT) by the wind farm of Annex I.



Fig. 18: Availability of reactive power absorption (inductive behaviour of the WT) by the wind farm of Annex I.

The calculus of availability is quite straightforward from CDF of the wind farms.

$$\Pr(\operatorname{Minimum} Q < q_{WT}) = \Pr(\operatorname{Power} < \sqrt{S_{max}^2 - q_{WT}^2})) -$$
(35)

- If
$$\left(\text{ Minimum } Q \Big|_{P_{WT}=0} < q_{WT} ; \Pr\left(Power < P_{undervoltage} \Big|_{operating region} \Big| (q_{WT}) \right) \right)$$

The optimum reactive power Q_{WT} , from voltage point of view, can be computed taking into account several node voltages, each one with its weighting factor.

$$Q_{WT} = \sum_{i=1}^{n} K_{Q_{WT},i} \frac{\Delta U_{\text{measured or}}}{\frac{\partial U_{node i}}{\partial Q_{WT}}}$$
(36)

XIII. EFFECT ON POWER LOSSES

The farm power output influence elements active and reactive power network losses. Power in shunt elements is voltage dependent in a non-linear way. Since the farm affects voltage only at nearby nodes and the main losses are in series elements, the influence of the farm in shunt losses will not be considered in this simplified approach.

Let's consider power losses $P_{losses, i}$ in a series element i that carry an apparent power $S_{element, i}$. The power injected by the farm would spread along the grid. Active and reactive power flows are quite decoupled and the farm power is approximately linearly distributed between parallel elements. Therefore, it is reasonable to use the following simplified model for the power loss in a grid element:

$$P_{\text{loss, i}} = \frac{R_{\text{i}}}{U_{\text{i}}} S_{\text{i}}^{2};$$

$$S_{\text{i}}^{2} \approx \left(P_{0,\text{i}} + k_{\text{P,i}} P_{\text{WT}}\right)^{2} + \left(Q_{0,\text{i}} + k_{\text{Q,i}} Q_{\text{WT}}\right)^{2}$$
(37)

 $P_{0,i}$ and $Q_{0,i}$ are the power flow at the elements when the turbines are disconnected. Approximate factors $k_{P,i}$ and $k_{Q,i}$ can be estimated simulating the network at maximum active and reactive power and computing the power flow difference at the element. The overall power loss with the aforementioned approximations would lead to a quadratic behaviour of net losses.

$$P_{\text{loss}} = \sum_{i} P_{\text{losses, }i} = P_{\text{loss}} |_{P_{\text{Wt}=0, Q_{\text{Wt}=0}}} + a_{\text{P}} P_{\text{WT}} + b_{\text{p}} P_{\text{WT}}^{2} + a_{\text{Q}} Q_{\text{WT}} + b_{\text{Q}} Q_{\text{WT}}^{2}$$
(38)

The five coefficients of (38) can be adjusted from the power flow losses in 5 different combinations of active P_{WT} and reactive Q_{WT} wind power.

If the network losses due to wind power are allocated mainly in elements electrically close to the wind farm, b_P and b_Q will have greater values. Thus, the relationship would be mainly quadratic with P_{WT} and Q_{WT} .

If wind power influence losses in elements mainly electrically far from the wind farm to vary, a_P and a_Q will have greater values. Thus, the relationship would be mainly linear with P_{WT} and Q_{WT} .

If network configuration or flows can change notably on high, medium and low load, the coefficients must be computed for those cases. Therefore, the reactive control of the wind farm might take into account the load classification at each time (a different control policy must be used depending on a scheduled load classification).

Reactive power losses also show an analogue relationship with P_{WT} and Q_{WT} .

XIV. UNCERTAINTY ANALYSIS

The uncertainty in the farm power output is due to:

- Adjustment of wind resource to a Weibull distribution.
- The uncertainty of the power curve.

0

- Simplistic model of the power curve with only two or four parameters.
- The wind farm speed characteristics are usually not well known and they depend on wind speed and direction. More over, the "undisturbed wind speed" should be estimated once the wind farm is in operation.
- Approximations done in the model of the grid (for example, considering U_0 constant).
- Availability of turbines and network.

The main source of uncertainty comes from the wind and the power curve. In case the performance matrix of the farm is available, the uncertainty can be notably decreased. The grid influence in power output is low since characteristics are usually well known and it is designed for high efficiency.

The availability of turbines is high due to scheduled maintenance and high reliability (the availability of the electrical network is even higher). However, the effect of events such as nearby short-circuits in some situations can trip many wind power stations. This is an example of very infrequent event but that can affect power quality because it concerns system stability in grids with high wind share.

The estimation of uncertainty is not easy. Apart form the ISO guide of uncertainty, GUM [26], all the power curve standards ISO 61400-12-1 [14], 61400-12-2 [27] and 61400-12-3 [18] include some annexes to help in uncertainty assessment.

The general procedure is to estimate the uncertainty of each component (i.e. power curve, wind distribution, etc). The sensitivity coefficients of individual uncertainty in the overall power output must be derived. Also, a model of propagation between uncertainties must be supposed. Should the uncertainties be uncorrelated, they partially cancels and the rooted sum of squares law should applied instead of the arithmetic sum of uncertainties. Sometimes, it is not clear which type of assumption is more adequate. In those cases, the assumptions can be classified as "conservative" or "optimistic". At the end, the uncertainty computed for several scenarios (optimistic, conservative, etc.) can be weighted by its likelihood or by an expert to obtain the expected uncertainty.

The uncertainty of the power output can be reduced using more detailed data. The process is roughly the same that has been presented here except that the majority of the computations must be done numerically. Moreover, the Monte Carlo method can be used to cope with detailed models. However, the increase of accuracy comes at the cost of a not so easy analysis of parameter sensitivity as in the analytic case.

Even though there is a small correlation of renewable energy and consumer load through the weather, this effect can be neglected in energy sources as wind and non-storage hydroelectric [28].

XV. CONCLUSIONS

This work shows a statistical model of wind farms and a methodology for adjusting its parameters. This model has been used to assess the grid impact of a wind farm reactive power during normal operation. Several reactive power control strategies are analyzed. The uncertainty of the final data due to the approximations made is studied. The accuracy can be increased if non-parametric models of farm power curve and wind resource is employed.

ANNEX I: EXAMPLE DATA

Power curve shown in figures:

 $w_{25\%}$ = 7,5 m/s; $w_{75\%}$ = 10,5 m/s; w_{cut-in} = 4 m/s; $w_{cut-off}$ = 25 m/s; $P_{nominal}$ = 1 p.u.; σ_{wf} = 1,5 m/s

Parameters of wind speed distribution: scale = $2 \mu_{wind} / \sqrt{\pi}$; shape = 2

Parameters of the wind farm:

 $\eta_{wf} = 0.93; \ \Delta w_{off} = 2 \text{ m/s}; \ w_{cut-off} = 25 \text{ m/s}; \ \sigma_{wf} = 1.5 \text{ m/s}; R_{series} = 0.03 \text{ p.u.}; \ X_{series} = 0.12 \text{ p.u}; \ G_{shunt} = 0.005 \text{ p.u.}; B_{shunt} = 0.01 \text{ p.u.}; \ R_{sc} = 0.02 \text{ p.u.}; \ X_{sc} = 0.18 \text{ p.u}; \mu_{wind} \text{ is assumed to be } 7 \text{ m/s if it is not stated}.$

Limits of reactive power generation:

 $S_{max} = 1 \text{ p.u.}; \quad \Delta Umax = 0.10 \text{ p.u.}$ at turbine converter.

BIOGRAPHIES

J Mur received his M.S. degree from the University of Zaragoza in 1997. He is working for towards his Ph.D. at the Zaragoza University. His research interests are in the field of renewable energy, reactive power control and integration of wind generation into electric grids. Currently, he is an Assistant Lecturer at the Electrical Department of the University of Zaragoza

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